A New Optimal Control Approach for Double Inverted Pendulum on Cart (DICP)

Asef. Zare, Saeed.Balochian, Mohammad.Reza .Arvan, Hossien.Balochian

***Abstract*—this paper presents a new control method for double inverted pendulum on cart problem. Firstly, model of system is presented. Secondly, it offers theorems and mathematical tools of optimal control of pendulum. Then, optimal control for double inverted pendulum on cart is designed. Finally, this algorithm and other methods are compared.**

***Keywords*—**optimal control-nonlinear programming-robot arm

I. INTRODUCTION

As a nonlinear under actuated plant, double inverted pendulum on a cart (DIPC) poses a challenging control problem. It seems to have been one of the attractive tools for testing linear and nonlinear control laws [5], [4], [3], [1]. Because Double inverted pendulum on cart is one of the complex nonlinear systems then**,** numerous papers used DIPC as a tested. Cited ones are merely an example. Nearly all works on pendulum control concentrate on two problems: pendulum swing up control design and stabilization inverted pendulum. This report, optimal nonlinear optimization problem is addressed: stabilize DIPC minimizing and an accumulative cost functional quadratic in dynamical equations of system, states and controls. The solution of nonlinear programming problem is control function such as piece wise control function. In [11] this optimal control problem is solved by a neural network, where NN estimate the vector state by using NN jacobian Matrix in each iteration.

All aforementioned methods have the same results and almost all of the existing methods of control is to take the pendulum to the equilibrium point and cannot follow another target. Now, the method (in this paper) of control System of DICP can have some more targets Simultaneously. For example the velocity or the end of state of cart and other conditions can be considered for physical system in the actual examination.

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II. Modeling

The DICP system is graphically showed in Fig.1. To derive its equation of motion, one of the possible ways is to use Lagrange equation:

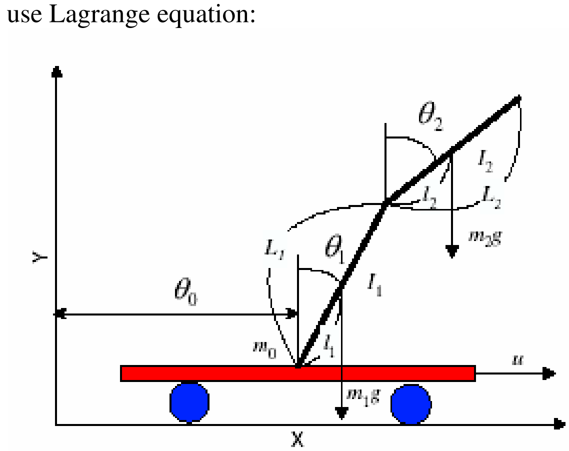

Figure1: double inverted pendulum on a cart

m	*Mass*
l_i	*Distance from a pivot joint to the i-th pendulum link Center of mass*
L_i	*Length of an i-th pendulum link*
θ_0	*Wheeled cart position*
θ_1, θ_2	*Pendulum angel*
I_i	*Moment of inertia of i-th pendulum link w.r.t its Center of mass*
g	*Gravity constant*
u	*Control force*
T	*Kinetic energy*
P	*Potential energy*
L	*Lagrangian*

$$\frac{d}{dt}\left(\frac{\partial}{\partial\dot{\theta}}\right)-\frac{\partial L}{\partial\theta}=Q \qquad (1)$$

Where: $L = T - P$ is a Lagrangian , Q is a vector of generalized forces (or moments) acting in the direction of generalized coordinates of θ and is not accounted in formulation of kinetic energy T and potential energy P. Kinetic and potential energies of the system are given by the sum of energies of its individual components (a wheeled cart and two pendulum):

$$T = T_0 + T_1 + T_2$$

$$P = P_0 + P_1 + P_2$$

Where:

$$T_0 = \tfrac{1}{2} m_0 \dot{\theta}_0^2$$

$$T_1 = \tfrac{1}{2} m_1 \left[\left(\dot{\theta}_0 + l_1 \dot{\theta}_1 \cos\theta_1\right)^2 + \left(l_1 \dot{\theta}_1 \sin\theta_1\right)^2 \right] + \tfrac{1}{2} I_1 \dot{\theta}_1^2 = \tfrac{1}{2} m_1 \dot{\theta}_0^2 + \tfrac{1}{2}\left(m_1 l_1^2 + I_1\right)\dot{\theta}_1^2 + m_1 l_1 \dot{\theta}_0 \dot{\theta}_1 \cos\theta_1$$

$$T_2 = \tfrac{1}{2} m_2 [\left(\dot{\theta}_0 + L_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2\right)^2 + (L_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2)^2] + \tfrac{1}{2} I_2 \dot{\theta}_2^2 = \tfrac{1}{2} m_2 \dot{\theta}_0^2 + \tfrac{1}{2} m_2 L_1^2 \dot{\theta}_1^2 + \tfrac{1}{2}(m_2 l_2^2 + I_2)\dot{\theta}_2^2 + m_2 L_1 \dot{\theta}_0 \dot{\theta}_1 \cos\theta_1 + m_2 l_2 \dot{\theta}_0 \dot{\theta}_2 \cos\theta_2 + m_2 L_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$P_0 = 0 \ , \qquad P_1 = m_1 g l_1 \cos\theta_1$$

$$P_2 = m_2 g (L_1 \cos\theta_1 + l_2 \cos\theta_2)$$

Thus the Lagrangian of the system is given by

$$L = \tfrac{1}{2}\left(m_0 + m_1 + m_2\right)\dot{\theta}_0^2 + \tfrac{1}{2}\left(m_1 l_1^2 + m_2 L_1^2 + I_1\right)\dot{\theta}_1^2 + \tfrac{1}{2}\left(m_2 l_2^2 + I_2\right)\dot{\theta}_2^2 + \left(m_1 l_1 + m_2 L_1\right)\cos(\theta_1)\dot{\theta}_0 \dot{\theta}_1 + m_2 l_2 \cos(\theta_2)\dot{\theta}_0 \dot{\theta}_2 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\dot{\theta}_1 \dot{\theta}_2 - \left(m_1 l_1 + m_2 L_1\right) g \cos(\theta_1) - m_2 l_2 g \cos(\theta_2)$$

Differentiating the Lagrangian by $\dot{\theta}$ and θ yields Lagrange equation (1) as:

$$\tfrac{d}{dt}\left(\tfrac{\partial}{\partial \dot{\theta}_0}\right) - \tfrac{\partial L}{\partial \theta_0} = u$$

$$\tfrac{d}{dt}\left(\tfrac{\partial}{\partial \dot{\theta}_1}\right) - \tfrac{\partial L}{\partial \theta_1} = 0$$

$$\tfrac{d}{dt}\left(\tfrac{\partial}{\partial \dot{\theta}_2}\right) - \tfrac{\partial L}{\partial \theta_2} = 0$$

Or explicitly:

$$u = (\textstyle\sum m_i)\ddot{\theta}_0 + (m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_1 + m_2 l_2 \cos(\theta_2)\ddot{\theta}_2 - (m_1 l_1 + m_2 L_1)\sin(\theta_1)\dot{\theta}_1^2 - m_2 l_2 \sin(\theta_2)\dot{\theta}_2^2$$

$$0 = (m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_0 + (m_1 l_1^2 + m_2 L_1^2 + I_1)\ddot{\theta}_1 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 + m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2 - (m_1 l_1 + m_2 L_1) g \sin\theta_1$$

$$0 = m_2 l_2 \cos(\theta_2)\ddot{\theta}_0 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 + (m_2 l_2^2 + I_2)\ddot{\theta}_2 - m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1^2 - m_2 l_2 g \sin\theta_2$$

Lagrange equation for the DICP system can be written in a more compact matrix form:

$$D(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Hu \qquad (2)$$

$$D(\theta) = \begin{pmatrix} d_1 & d_2 \cos\theta_1 & d_3 \cos\theta_2 \\ d_2 \cos\theta_1 & d_4 & d_5 \cos(\theta_1 - \theta_1) \\ d_3 \cos\theta_2 & d_5 \cos(\theta_1 - \theta_2) & d_6 \end{pmatrix} \qquad (3)$$

$$C(\theta,\dot{\theta}) = \begin{pmatrix} 0 & -d_2 \sin(\theta_1)\dot{\theta}_1 & -d_3 \sin(\theta_2)\dot{\theta}_2 \\ 0 & 0 & d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_2 \\ 0 & -d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_1 & 0 \end{pmatrix} \qquad (4)$$

$$G(\theta) = \begin{pmatrix} 0 & -f_1 \sin\theta_1 & -f_2 \sin\theta_2 \end{pmatrix}^T \qquad (5)$$

$$H = (1,0,0)^T$$

Assuming that centers of mass of pendulums are in the geometrical center of the links, which are solid rods, we have: $l_i = L_i/2, I_i = m_i L_i^2/12$ .Then for the elements of matrices $D(\theta)$, $C(\theta,\dot{\theta})$, and $G(\theta)$ we get:

$$d_1 = m_0 + m_1 + m_2$$

$$d_2 = m_1 l_1 + m_2 L_1 = \left(\tfrac{1}{2} m_1 + m_2\right) L_1$$

$$d_3 = m_2 l_2 = \tfrac{1}{2} m_2 L_2$$

$$d_4 = m_1 l_1^2 + m_2 l_1^2 + I_1 = \left(\tfrac{1}{3} m_1 + m_2\right) L_1^2$$

$$d_5 = m_2 L_1 l_2 = \tfrac{1}{2} m_2 L_1 L_2$$

$$d_6 = m_2 l_2^2 + I_2 = \tfrac{1}{3} m_2 L_2^2$$

$$f_1 = \left(m_1 l_1 + m_2 L_1\right) g = \left(\tfrac{1}{2} m_1 + m_2\right) L_1 g$$

$$f_2 = m_2 l_2 g = \tfrac{1}{2} m_2 L_2 g$$

III. Mathematical Tools

Assuming that nonlinear system equation in state space is such as:

$$\dot{X} = g(X,u) \qquad\qquad \textit{Problem (1)}$$
$$X(a) = X_a \qquad X(b) = X_b$$

Where $X = [x_1, x_2, ..., x_n]^T$ is state vector, if function F is defined such as:

$$F(x_1(t), x_2(t), ..., x_n(t)) = \left\| \dot{X} - g(X,u) \right\|$$

That $\|.\|$ is 2-norm on R^n, that is defined by:

$$\|y\| = \sum_{i=1}^{n} |y_i|^2 \ .$$

Then existence of solution of problem (1) transfers to a calculus of variation problem such as:

$$Min \quad J(x_1(0), x_2(0), ... x_n(0)) = \int_a^b F(x_1(t),$$

$$x_2(t), \ldots, x_n(t), u(t)).dt$$

$S.T$ problem(2)

$$x_1(a) = x_{a1} \quad x_2(a) = x_{a2} \ldots x_n(a) = x_{an}$$

$$x_1(b) = x_{b1} \quad x_2(b) = x_{b2} \ldots x_n(b) = x_{bn}$$

Theorem 1: Problem (1) has a solution if and only if optimal value of cost function of problem (2) is zero.

Proof: If the control function, $u(t)$ is a piece wise continues on $[a,b]$ then: $F(x_1(t), x_2(t), ..., x_n(t), u(t))$ is a real nonnegative function. Therefore the Cost function of problem (2) in optimal case is zero and $x_1^*(t), x_2^*(t), ..., x_n^*(t), u^*(t)$ are functions that also satisfy in problem (1). Then for optimal solution $\int_a^b F(x_1^*(t), x_2^*(t), ..., x_n^*(t), u^*(t)).dt = 0$ and because F is a continuous and nonnegative function, Then $\dot{X}^* - g(\overset{*}{X}, \overset{*}{u}) = 0$ or $F(x_1^*(t), x_2^*(t), ..., x_n^*(t), u^*(t)) = 0$ almost every whe-re on $[a,b]$ and also:

$$x_1^*(t_a) = x_{a1} \quad x_2^*(t_a) = x_{a2} \ldots x_n^*(t_a) = x_{an}$$

$$x_1^*(t_b) = x_{b1} \quad x_2^*(t_b) = x_{b2} \ldots x_n^*(t_b) = x_{bn}$$

Therefore $x_1^*(t), ..., x_n^*(t), u^*(t)$ is a solution of problem (1). Suppose $x_1(t), x_2(t), ..., x_n(t), u(t)$ be answer of problem (1), we can show easily that $x_1(t), x_2(t), ..., x_n(t), u(t)$ is an optimal solution of problem (2) and optimal value of cost function will be zero.

IV. Discretization

We partition interval [a, b] to N equal parts and define $\Delta = \dfrac{b-a}{N}$ let:

$$x_{ij} = x_i(t_j) \quad i = 1, ..., n \quad j = 1, ..., N+1$$

Because N is large, then $\dot{x}_{ij}$ can be approximated such as:

$$\dot{x}_{ij} \approx \frac{x_{ij+1} - x_{ij}}{\Delta} \; i = 1, ..., n \quad j = 1, ..., N+1$$

Also control function can be approximated as:

$$u_j = u_l(t_j) \quad l = 1, ..., m \quad j = 1, ..., N+1$$

So general problem can be written such as:

$$\dot{X} = g(X, u) \Rightarrow \dot{x}_i = g(x_{ij}, u_{lj})$$

And with define:

$$f_i = \frac{x_{ij+1} - x_{ij}}{\Delta} - g(x_{ij}, u_{lj})$$

$$i = 1, ..., n \quad j = 1, ..., N+1$$

Therefore using integral definition method can change this integral to discrete form. Then:

$$Min \quad J = \frac{1}{n}\sum_{i=1}^{n} \left\| \dot{X} - g(X_{ij}, u_j) \right\| \xrightarrow{\equiv}$$

$$Min \quad J = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{N+1} |f_i|^2 \qquad (6)$$

$s.t:$

$$f_1 = \frac{x_{1j+1} - x_{1j}}{\Delta} - g(x_{1j}, u_j)$$

$$\vdots$$

$$f_n = \frac{x_{nj+1} - x_{nj}}{\Delta} - g(x_{nj}, u_j)$$

$$x_{1a} = \alpha_1 \qquad x_{1b} = \beta_1$$

$$x_{2a} = \alpha_2 \qquad x_{2b} = \beta_2$$

$$\vdots \qquad \vdots$$

$$x_{na} = \alpha_n \qquad x_{nb} = \beta_n$$

Finally using theorem 1, the existence of solution of problem (1) is equal to solving nonlinear programming (N.L.P) problem (6).

Theorem (2): If optimal cost function of nonlinear programming problem (6) is zero, so $\left(\overset{*}{X}, \overset{*}{u}\right)$ is a solution for problem (1)

Proof: According to structure of $\overset{*}{X}(t), \overset{*}{u}(t)$ that was connected pieces of function with quantity of proper target of problem (2) is zero. Therefore according to theorem (1) the pair of $\overset{*}{X}(t)$, $\overset{*}{u}(t)$ is the answer for problem (1). According to equation (2), (3), (4), (5), we can change dynamic equation of DICP system to the state space. Firstly, state variable is defined such as:

$$x_1 = \theta_0 \qquad x_2 = \dot{\theta}_0 \qquad x_3 = \theta_1$$

$$x_4 = \dot{\theta}_1 \qquad x_5 = \theta_2 \qquad x_6 = \dot{\theta}_5$$

Then, the equations of state space system are represented as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_5 = x_6 \\ d_1\dot{x}_2 + d_2\dot{x}_4 \cos(x_3) + d_3\dot{x}_6 \cos(x_5) \\ -d_2 x_4^2 \sin(x_3) - d_3 x_6^2 \sin(x_5) = u \\ d_2\dot{x}_2 \cos(x_3) + d_4\dot{x}_4 + d_5\dot{x}_6 \cos(x_3 - x_5) \\ +d_5 x_6^2 \sin(x_3 - x_5) - f_1 \sin(x_3) = 0 \\ d_3\dot{x}_2 \cos(x_5) + d_5\dot{x}_4 \cos(x_3 - x_5) \\ +d_6\dot{x}_6 - d_5 x_4^2 \sin(x_3 - x_5) - f_2 \sin(x_5) = 0 \end{cases}$$

Now, according to theorem (1) the problem can be changed to an optimal nonlinear control problem, such as:

$$\begin{aligned} Min \quad J = \int_0^1 \Big[& |\dot{x}_1 - x_2|^2 + |\dot{x}_3 - x_4|^2 + |\dot{x}_5 - x_6|^2 \\ & + |d_1\dot{x}_2 + d_2\dot{x}_4 \cos(x_3) + d_3\dot{x}_6 \cos(x_5) \\ & - d_2 x_4^2 \sin(x_3) - d_3 x_6^2 \sin(x_5) - u|^2 + \\ & |d_2\dot{x}_2 \cos(x_3) + d_4\dot{x}_4 + d_5\dot{x}_6 \cos(x_3 - x_5) \\ & + d_5 x_6^2 \sin(x_3 - x_5) - f_1 \sin(x_3)|^2 + \\ & |d_3\dot{x}_2 \cos(x_5) + d_5\dot{x}_4 \cos(x_3 - x_5) + \\ & d_6\dot{x}_6 - d_5 x_4^2 \sin(x_3 - x_5) - f_2 \sin(x_5)|^2 \Big] dt \end{aligned} \tag{7}$$

$s.t.:$

$x_{10}(0) = a_1$	$x_{11}(1) = \beta_1$
$x_{20}(0) = a_2$	$x_{21}(1) = \beta_2$
$x_{30}(0) = a_3$	$x_{31}(1) = \beta_3$
$x_{40}(0) = a_4$	$x_{41}(1) = \beta_4$
$x_{50}(0) = a_5$	$x_{51}(1) = \beta_5$
$x_{60}(0) = a_6$	$x_{61}(1) = \beta_6$

Where $a_1, a_2, a_3, a_4, a_5, a_6, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ are the initial and final conditions of nonlinear system and are constant. With a suitable change of variable, we can transform the interval of integration $[0, \infty)$ to bounded interval [0, 1) [7], [8], [9], [10]. Then, we can divide the interval of integration to N (=50) sub equal intervals. Finally, we have the nonlinear programming problem as:

$$\begin{aligned} Min \quad J = \frac{1}{50}\sum_0^{49} \Big[& |n(x_{1i+1} - x_{1i}) - x_{2i}|^2 \\ & + |n(x_{3i+1} - x_{3i}) - x_{4i}|^2 + |n(x_{5i+1} - x_{5i}) - x_{6i}|^2 \Big] \\ & + |d_1 n(x_{2i+1} - x_{2i}) + d_2 n(x_{4i+1} - x_{4i}) \cos(x_3) \\ & + d_3 n(x_{6i+1} - x_{6i}) \cos(x_5) - d_2 x_4^2 \sin(x_3) \\ & - d_3 x_6^2 \sin(x_5) - u|^2 + |d_2 n(x_{2i+1} - x_{2i}) \cos(x_3) \\ & + d_4 n(x_{4i+1} - x_{4i}) + d_5 n(x_{6i+1} - x_{6i}) \cos(x_3 - x_5) + \\ & d_5 x_6^2 \sin(x_3 - x_5) - f_1 \sin(x_3)|^2 + \\ & |d_3 n(x_{2i+1} - x_{2i}) \cos(x_5) + d_5 n(x_{4i+1} - x_{4i}) \cos(x_3 - x_5) \\ & + d_6 n(x_{6i+1} - x_{6i}) - d_5 x_4^2 \sin(x_3 - x_5) - f_2 \sin(x_5)|^2 \Big] \end{aligned} \tag{8}$$

$s.t:$

$x_{10}(0) = a_1$	$x_{11}(1) = \beta_1$
$x_{20}(0) = a_2$	$x_{21}(1) = \beta_2$
$x_{30}(0) = a_3$	$x_{31}(1) = \beta_3$
$x_{40}(0) = a_4$	$x_{41}(1) = \beta_4$
$x_{50}(0) = a_5$	$x_{51}(1) = \beta_5$
$x_{60}(0) = a_6$	$x_{61}(1) = \beta_6$

To solve the above nonlinear programming problem, we can use any first condition and gain 50 point for each variable of states and control ($x_1, x_2, x_3, x_4, x_5, x_6, u$) in the interval of [0,1). Then, using a curved fitting method, we can obtain control function as a continuous function according to theorem 2. One of the advantages of this method is that it can consider multi-objectives such as bringing the pendulums to equilibrium of zero state, and the cart can move till a limited point. In other words, there are limitations on position and velocity of cart. Although in [1],[2],[3] the above mentioned state is not applicable, it is possible for nonlinear programming problem if we add a constraint to (8).

The only problem for it is its bulky calculations or for controller design we need to solve nonlinear programming problem with an initial condition for the very design. We can solve the problems with one of the following methods.

A. *The first method*

Although the existed methods in DICP system all have bulky calculations, nonlinear programming software such as LINGO.10.0 or WHAT BEST can do the calculations in a fraction of second. Therefore, this algorithm can be applicable in an online format.

B. The second method:

In this method, we first solve nonlinear programming problem (8) with different initial conditions in which control and state functions are determined as a continuous function. Then, the pairs of initial conditions and optimal control are fed into an MLP neural network to be trained. Finally, using characteristic of MLP neural networks, we can obtain control function which is close to optimal control for each initial condition. So that, this method is the basis of neural network controller design for DICP system.

V. Simulation

Firstly, we solve nonlinear programming (8) in stead of some of initial conditions for simulation parameters shown in table 1. The variables $x_1, x_2, x_3, x_4, x_5, x_6$ are the place of cart, motion velocities of the bottom pendulum, position and velocities, top pendulum position and its velocities.

As we see, the control signals in the method do not have oscillation that makes it practical with electric motors, while in other methods control signal has high frequency oscillation which its application is problematic. Hence, as other method, the design is based on linear zed model that brings about the control function get away from optimal control law. Although, this problem cause state functions especially in position of the bottom and the top pendulum oscillate at zero with high frequency, in our method the convergence of state and control functions are very quick. Moreover, in the new method, the initial conditions not only are not necessarily in the zero point but also, can be very far from equilibrium point. It is to say that, in other method the initial condition should be close to equilibrium point because they use linear zed system.

Table 1: simulation parameter

Parameter	Value
m_0	1.5 kg
m_1	0.5 kg
m_2	0.75 kg
L_1	0.5 m
L_2	0.75 m
Δt	0.02 s

VI. Conclusion

This paper presented a new approach for solving of DIPC problem. Firstly, Control problem was transformed to a nonlinear optimal control problem. Then, this problem is changed to a nonlinear programming problem that by its solving control function is determined as a piece vise constant function.

In this paper, we used an exact nonlinear model as other methods used a linearazed model and their applications were limited. Also, this method uses only one electromotor for optimal control in which other conditions in the actual design e.g. limited oscillation in control signal and maximum motion of cart or speed of motor are considered.

$\theta_1 = -10^\circ, \theta_2 = 10^\circ$

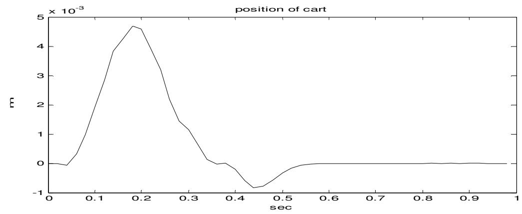

Fig. 2 position of cart

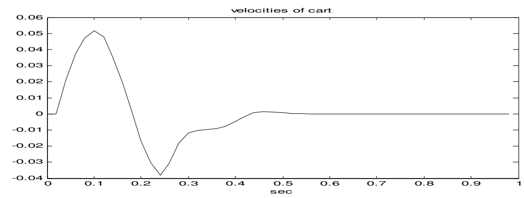

Fig. 3 velocities of cart

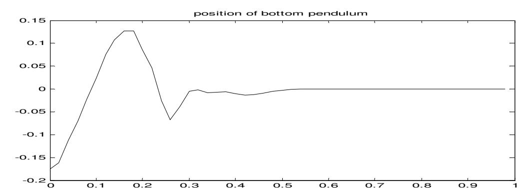

Fig.4 position of bottom pendulum

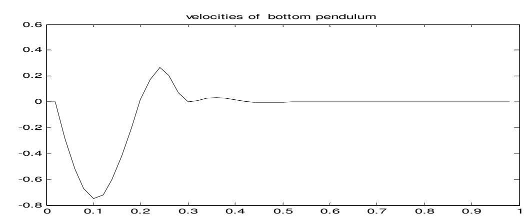

Fig.5 velocities of bottom pendulum

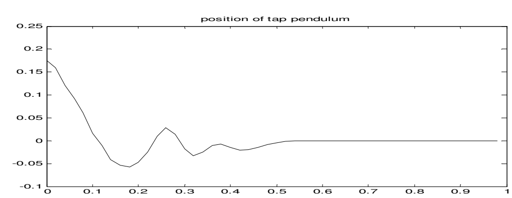

Fig.6 position of top pendulum

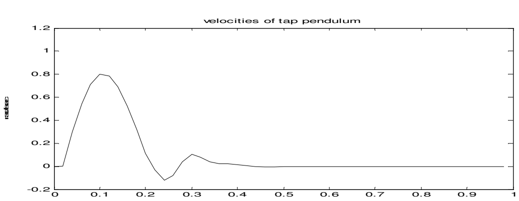

Fig.7 velocities of top pendulum

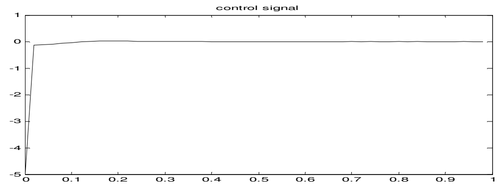

Fig.8 control signal

$$\theta_1 = -10^\circ, \theta_2 = 40^\circ$$

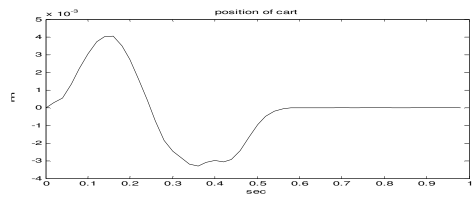

Fig. 9 position of cart

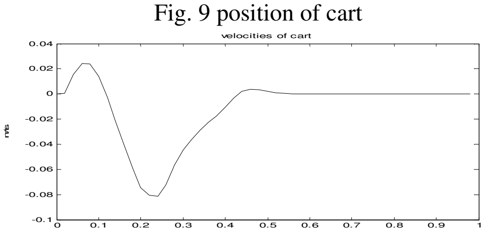

Fig. 10 velocities of cart

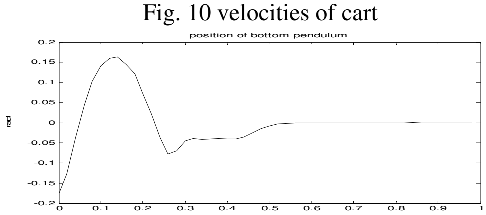

Fig.11 position of bottom pendulum

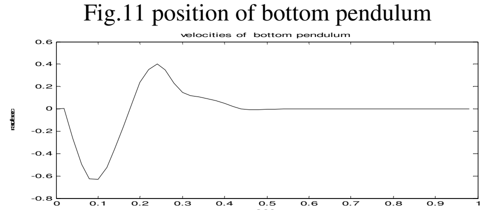

Fig.12 velocities of bottom pendulum

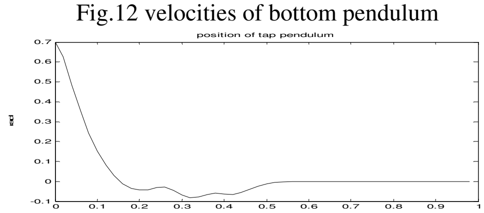

Fig.13 position of top pendulum

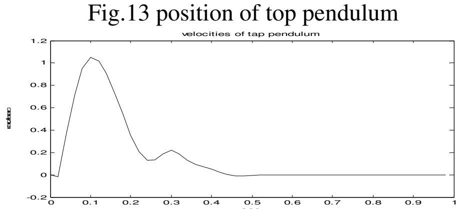

Fig.14 velocities of top pendulum

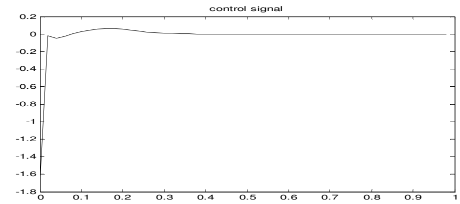

Fig.15 control signal

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