ANALYSIS OF THE CELL AVERAGING CFAR IN WEIBULL BACKGROUND USING A DISTRIBUTION APPROXIMATION

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ABSTRACT

In this work, we study the performance of the cell averaging (CA) CFAR processor in the presence of uniform Weibull clutter. The major issue in this analysis is the unavailability of the distribution of the sum of independent and identically distributed (i.i.d.) Weibull random variables. We use an approximation to the distribution of the sum of i.i.d. Weibull random variables for the theoretical analysis of the CA-CFAR processor. The detection performance analysis has been done using both numerical solutions of the derived equations and Monte-Carlo simulations. The CFAR loss has also been worked out. The results show that the CA-CFAR processor has acceptable performance for a wide range of the shape parameter, accompanied by its added advantage of simplicity of implementation for real-time applications. The performance of the CA-CFAR detector can be used as a yardstick for the analysis of other CFAR processors in the presence of Weibull clutter.

1. INTRODUCTION

Modern radars use constant false alarm rate (CFAR) processing for the automatic detection of targets that are immersed in clutter and receiver thermal noise. The underlying principle for CFAR processors is to set a threshold by fixing the false alarm probability (P_{FA}) at a certain level while maximizing the probability of detection (P_D). For a stationary background, the optimum fixed threshold is given by the Neyman-Pearson theorem [1]. But for a non-stationary background, where the background power levels vary, the threshold no longer remains constant and has to adapt according to the varying background parameters in order to maintain the false alarm rate at the desired level. The simplest of the CFAR processors is the cell-averaging (CA) CFAR [2] processor that estimates the background power by averaging the background cells and sets the threshold accordingly. Variations of the CA-CFAR processor include the greatest-of (GO) CFAR [2] and the smallest-of (SO) CFAR [2] processors that have better performance than the CA-CFAR processor in non-homogeneous backgrounds. Robust CFAR algorithms the ordered statistic (OS) CFAR [3] and algorithms with the minimum estimation variance like the maximum-likelihood (ML) CFAR [4] have been studied thoroughly in the literature.

At high-resolutions (i.e. less than 15 meters [5]) the background clutter statistics exhibit deviation from the Gaussian nature. The radar returns become spikier and the false alarm level increases which results in poor performance. This phenomenon is more pronounced at low grazing angles [6]. An additional skewness parameter is needed in the probability model to successfully describe the clutter in this situation. Various probability distributions have been used to model clutter returns for radars operating in a variety of backgrounds like ground, sea, etc. Popular models for radar clutter are the Weibull, lognormal, K-, and gamma distributions. The Weibull distribution, first proposed as a model for radar clutter in [7] is known to give the best fit for land clutter [6]. The performance of the CA-CFAR processor is difficult to analyze in Weibull background because the distribution of the sum of Weibull variates is not known in the closed form. Alternative CFAR processors like the optimal Weibull (OW) CFAR [8], the ML-CFAR [4], and the OS- CFAR [9], have been studied thoroughly, but a serious drawback of these alternative schemes is their complexity of implementation for real-time applications.

In the present work we analyze the performance of the CA- CFAR detector using an approximate model for the sum of Weibull variates. The expressions for the detection probability and CFAR loss of the CA-CFAR processor, operating in the Weibull background, have been derived. The results show that the CA-CFAR processor has satisfactory performance for a wide range of the shape parameter implying the suitability of the CA-CFAR processor for a wide range of operating environments. The performance analysis done here can be used mainly as a yardstick in assessing other CFAR detectors in the presence of Weibull clutter. The present work is organized as follows. Section 2 gives a brief review of CFAR processors and the Weibull distribution. Section 3 gives the major results derived in the present work for the CA-CFAR processor operating in the Weibull background, the detection performance curves and CFAR loss. Finally, the conclusions are drawn in section 4.

2 CFAR PROCESSING AND THE WEIBULL DISTRIBUTION

This section gives a brief review of CFAR processing, the CA-CFAR processor, and the approximate distribution analysis of the CA-CFAR processor. The equation for calculating the threshold of the CA-CFAR processor is also presented.

2.1 CFAR Processing

The block diagram for a general CFAR processor is shown in Figure 1 [8].

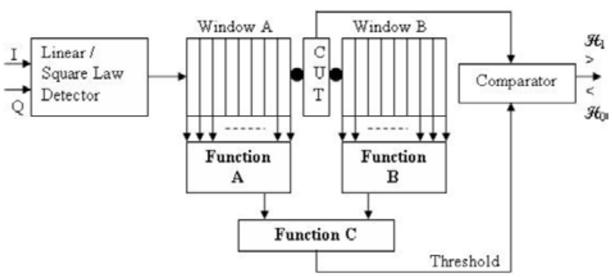

Figure 1 – Generic CFAR Block Diagram

The baseband I and Q signals are first passed through a detector and the resulting samples are gathered in the form of a batch of size N+1. The sample in the middle is termed the cell under test (CUT), and it divides the whole batch into two halves, termed "Window A" and "Window B". The samples from each of the two windows enter some processing units, labeled "Function A", "Function B" and "Function C". What these blocks actually do depends on the type of CFAR processor. The output of processing the background cells is a threshold value with which the CUT is compared and a decision for the presence or absence of a target is made.

For the cell-averaging CFAR processor, Function A and B are merged and the average of all the background cells is computed. The resulting statistic is multiplied by a constant scaling factor depending on the false alarm probability (P_{FA}) to compute the threshold. This is shown in Figure 2 [2].

2.2 The CA-CFAR in Weibull distributed clutter

The Weibull distribution is a two-parameter distribution, and is used for statistical modeling in many branches of sciences and engineering. The Weibull probability density function (PDF) is given by [10]:

$$f(x) = bcx^{b-1}\exp(-cx^{b}) \tag{1}$$

where *'b'* is the shape parameter and *'c'* is the scale parameter.

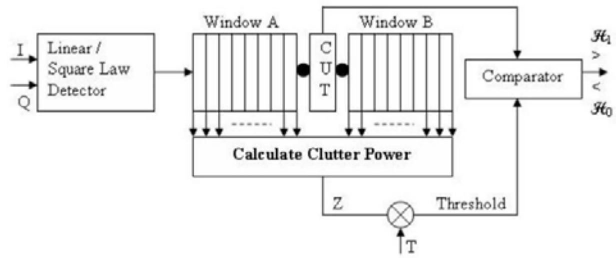

Figure 2 – Block Diagram of the CA-CFAR Processor

The Weibull cumulative density function (CDF) is given by

$$F(x) = 1 - \exp\left(-cx^{b}\right); x > 0 \tag{2}$$

For the cell-averaging CFAR processor, the statistic Z, representing the average clutter power, shown in Figure 2, is computed as the sum of the background samples. That is:

$$Z = \sum_{i=1}^{N} X_i \tag{3}$$

The exact PDF of the statistic Z is that of the sum of Weibull distributed random variables, which is not known in closed form. Recently, [10] has suggested a rational and closely accurate formula for the distribution of the sum of independent and identically distributed (i.i.d.) Weibull random variables. The PDF is a type of a generalized Gamma distribution given by [10]:

$$f_N(z) = \frac{rbc^N}{\Gamma(N)}(rz)^{Nb-1}exp[-c(rz)^b] \tag{4}$$

where,

$$r = \frac{\Gamma(N+1/b)}{N\Gamma(N)\Gamma(1+1/b)} \tag{5}$$

The parameter 'r' can be considered to be a scale parameter, so that the first moment (the mean) of the distribution (4) equals the mean of the sum of N Weibull distributed random variables. Using (4) we can write the false alarm probability as [10]:

$$\begin{aligned} P_{fa} &= \int_0^\infty P(CUT > TZ) f_N(z)dz \\ &= \int_0^\infty \exp[-c(Tz)^b] f_N(z)dz \end{aligned} \tag{6}$$

Solving the integral we get

$$P_{fa} = \frac{1}{[1+(T/r)^b]^N} \tag{7}$$

We now use the equations (4) and (7) for further analysis of the CA-CFAR processor in Weibull clutter.

3 DETECTION PROBABILITY AND CFAR LOSS

In this section we summarize the important equations and simulation results obtained for the CA-CFAR processor operating in uniform Weibull background. We start with a discussion on the P_D and its calculation with certain assumptions about the clutter/target statistics and then present the detection performance curves. After that, the calculations for the CFAR loss and quantification of the CFAR loss for various shape parameters will be presented.

3.1 Probability of Detection

The detection probability for CA-CFAR processor is given by:

$$P_D = \int_0^\infty P(CUT > TZ) f_N(z) dz \quad (8)$$

where $f_N(z)$ is the PDF of the sum of i.i.d. Weibull random variables given in (4) and

$$P(CUT > T) = \int_T^\infty f_{CUT}(x) dx \quad (9)$$

$f_{CUT}(x)$ is the PDF of the sum of target and clutter returns. For our analysis, we assume a Rayleigh fluctuating target with mean power B^2_{TGT}. The PDF of the sum of Weibull and Rayleigh variates is also not known in the closed form. We therefore assume the clutter to be Rayleigh distributed in the CUT with the same mean power as that of the Weibull clutter. The mean clutter power is computed as the second moment of the Weibull distribution,

$$P_{clutter} = E(X^2) \quad (10)$$

where the expectation is taken according to the Weibull distribution given in (1). Therefore,

$$P_{clutter} = c^{-2/b}\Gamma\left(1 + \frac{2}{b}\right) \quad (11)$$

The PDF of the CUT will be the PDF of the sum of target and clutter samples, both of which are assumed to follow the Raleigh distribution. Therefore, the CUT will also follow the Raleigh distribution, with mean power equal to the sum of the target and clutter powers.

$$\begin{aligned} f_{CUT}(x) &= \frac{2x}{c^{-2/b}\Gamma\left(1+\frac{2}{b}\right) + B^2_{TGT}} \quad (12) \\ &\times \exp\left[\frac{-x^2}{c^{-2/b}\Gamma\left(1+\frac{2}{b}\right) + B^2_{TGT}}\right] \end{aligned}$$

Defining the signal-to-clutter ratio (SCR) as

$$SCR = \frac{B^2_{TGT}}{c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)} \quad (13)$$

we can write (12) as

$$\begin{aligned} f_{CUT}(x) &= \frac{2x}{(1+SCR)c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)} \quad (14) \\ &\times \exp\left[\frac{-x^2}{(1+SCR)c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)}\right] \end{aligned}$$

Substituting (14) in (9) we get,

$$P(CUT > TZ) = \exp\left[\frac{-(Tz)^2}{(1+SCR)c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)}\right] \quad (15)$$

Substituting (4) and (15) in (8) we get,

$$\begin{aligned} P_D &= \int_0^\infty \exp\left[\frac{-(Tz)^2}{(1+SCR)c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)}\right] \quad (16) \\ &\times \frac{rbc^N}{\Gamma(N)}(rz)^{Nb-1}\exp[-c(rz)^b]\, dz \end{aligned}$$

Or,

$$\begin{aligned} P_D &= \frac{r^{Nb}bc^N}{\Gamma(N)}\int_0^\infty z^{Nb-1} \quad (17) \\ &\times \exp\left[-\left\{\frac{(Tz)^2}{(1+SCR)c^{-2/b}\Gamma\left(1+\frac{2}{b}\right)} + c(rz)^b\right\}\right] dz \end{aligned}$$

This integral cannot be evaluated in the closed form. However, for the special case of b=2, (17) simplifies to,

$$P_D = \left[1 + \frac{T^2}{r^2(1+SCR)}\right]^{-N} \quad (18)$$

where *'r'* is defined in (5). Except for this special case the analytic solution to (17) is not available and we have to resort to the numerical solution. Equation (17) is an approximation to the actual P_D because of the Rayleigh clutter assumption despite the fact that the clutter follows the Weibull distribution in the CUT. The accuracy of this approximation is calculated using the method proposed in [4]. Figure 3 illustrates the accuracy of the approximation where we have compared the P_D calculated using (17) with the Monte-Carlo simulation in which the CUT contained Weibull clutter and a Rayleigh target, for the CA-CFAR.

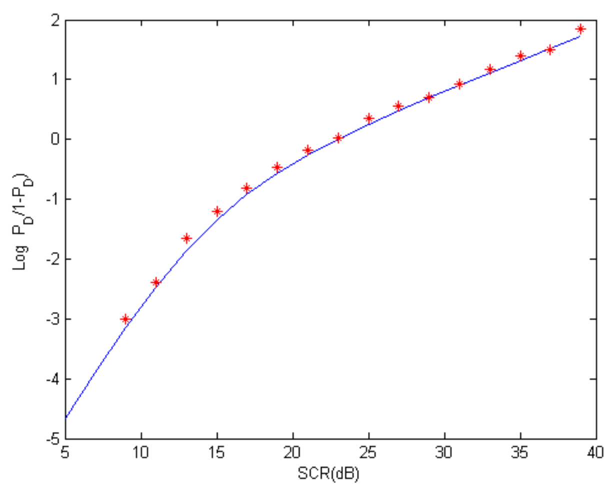

Figure 3 – Comparison of the theoretical approximation in (17) and Monte-Carlo simulation for the detection probability as a function of SCR ($P_{FA} = 10^{-5}$, N = 16, b = 1)

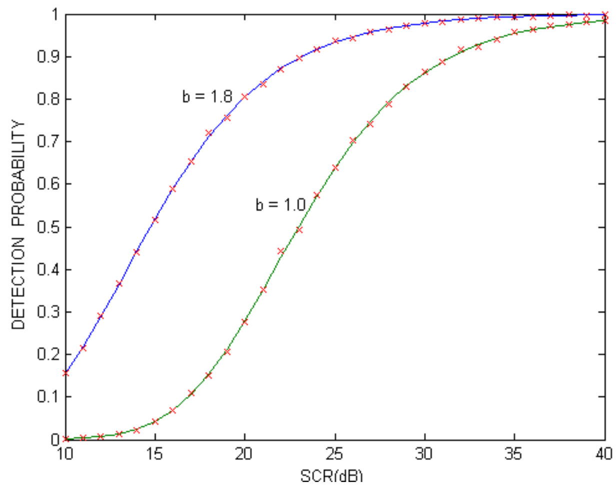

Figure 4 – Comparison of the numerical solution to (17) with Monte-Carlo simulations ($P_{FA} = 10^{-5}$, N = 16)

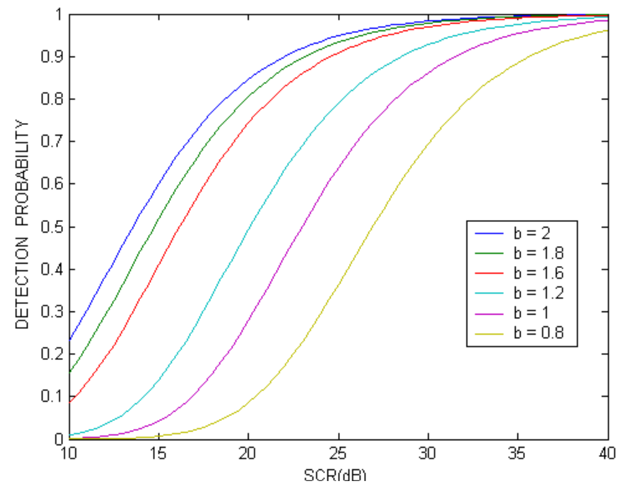

Figure 5 – Detection performance of the CA-CFAR in Weibull Clutter ($P_{FA} = 10^{-5}$, N = 16)

The solid line shows the theoretical approximation in which the CUT is assumed to contain a Rayleigh target plus the Rayleigh background. The dots represent the results of Monte-Carlo simulation in which the CUT contained the target plus correct Weibull background. For the simulation, the window size N = 16, b = 1, and $P_{FA} = 10^{-5}$. Figure 3 shows that (17) is sufficiently accurate in describing the probability of detection of the CA-CFAR when the background is Weibull distributed, has any shape parameter b, SCR >> 1 and the target is Rayleigh fluctuating.

The Monte-Carlo simulations give a better idea of the loss due to the approximate distribution used in our analysis. Although the scaling factor *'T'* is computed using (7), the statistic Z is an actual sum of Weibull variates. The numerical solutions to (17) and results from Monte-Carlo simulations have been plotted in Figure 4. The solid lines show the numerical solution of (17) whereas the crosses show results from running Monte-Carlo simulations. Based on the close agreement between the two, we can conclude that (17) indeed gives a fairly good estimate of the actual P_D. Further numerical plots for equation (17) are shown in Figure 5 for N = 16 and $P_{FA} = 10^{-5}$ for several values of the shape parameter b. As expected, the P_D decreases for the same SCR as the shape parameter b decreases.

3.2 CFAR Loss

CFAR loss is defined as the ratio between the SCR required to achieve a specified P_D and P_{FA}, and the SCR of the non-CFAR case in which the clutter level is known and the threshold is constant,

$$CFAR\ Loss = \frac{SCR(P_{fa}, P_D, b, N)}{SCR_{opt}(P_{fa}, P_D, b)} \tag{19}$$

Designating the fixed threshold as t_∞, the following relationships hold in the non-CFAR case (assuming Weibull clutter and Rayleigh target),

$$P_{fa} = \exp\left[-c\, t_\infty^b\right] \tag{20}$$

$$P_D = \exp\left[\frac{-(c^{1/b}t_\infty)^2}{(1+SCR)\Gamma(1+\frac{2}{b})}\right] \tag{21}$$

From (20) and (21) we obtain the relationship between SCR, P_{FA} and P_D for the non-CFAR case as,

$$SCR_{opt} = \frac{(ln(\frac{1}{P_{fa}}))^{2/b}}{(ln(\frac{1}{P_D}))\Gamma(1+\frac{2}{b})} - 1 \tag{22}$$

where SCR is calculated for a given P_{FA}, P_D, b, N by iteratively solving (17). With the help of the equations given above, the CFAR loss is plotted in Figure 6 for a $P_{FA}=10^{-5}$ and N=16.

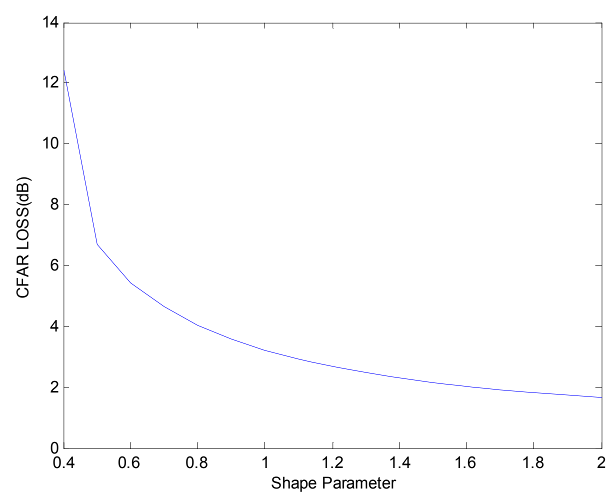

Figure 6 – CFAR loss of CA-CFAR against the shape parameter ($P_{FA} = 10^{-5}$, N = 16)

From Figure 5 we see that in order to maintain P_D=0.5 as b drops from 2 to 0.8, an increase of about 13dBs of SCR is necessary. The increase is constructed from about 2.4dB additional CFAR loss (Figure 6) and a balance of 10.6dB increase that would have been required by a fixed-threshold detector. The CFAR loss increases rapidly for values of the shape parameter less than 0.5. But in most practical scenarios the value of b is greater than 0.5 [8].

4 CONCLUSIONS

In this paper, we analyzed the traditional cell-averaging CFAR processor in uniform Weibull background. The theoretical analysis was done using a newly proposed approximation to the distribution of the sum of Weibull distributed data. The detection performance analysis and CFAR loss for the CA-CFAR were presented, using both the numerical solution and Monte-Carlo simulations. The present work can be extended to the non-uniform clutter case as well. The analysis presented can be used mainly as a yardstick in assessing the performance of other CFAR detectors. The CA-CFAR processor is very easy to implement and the results shown here confirm that the CA-CFAR processor can also be used in radars operating in the Weibull background.

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