Performance Comparison of Wavelet Packet Modulation and OFDM for Multipath Wireless Channel

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***Abstract*— In comparison to Orthogonal Frequency Division Multiplexing (OFDM), Wavelet Packet Modulation (WPM) offers much lower side lobes in transmitted signal, which reduces inter-carrier interference (ICI) and narrowband interference (NBI). It is also spectrally efficient since it does not utilize cyclic prefix (CP), nevertheless, it requires an efficient equalization technique to counter the inter symbol interference (ISI) and ICI created by the channel. In this paper, performance comparison of OFDM and WPM for several multipath wireless channels is presented. We propose a novel application of zero-forcing (ZF) and minimum mean square error (MMSE) algorithms as time-domain channel equalization techniques for WPM systems. It is shown that, for a multipath wireless channel, WPM using MMSE-equalizer has a better bit error rate (BER) performance as compared to ZF-equalizer in WPM, as well as OFDM system.**

Index Terms— Wavelet Packet Modulation, Minimum Mean Square Error Equalization, Zero-Forcing Equalization.

I. Introduction

OFDM is a Discrete Fourier Transform (DFT) based multi-carrier modulation (MCM) scheme. It utilizes CP in guard band interval to cancel ISI caused by multipath wireless channel [1]. However, CP utilizes extra bandwidth which makes OFDM spectrally inefficient. Moreover, OFDM uses rectangular pulse shape of sinusoidal carriers, having high side lobes [2]. High side-lobes in transmitted signal increases OFDM system's sensitivity to ICI and NBI [2]. Therefore, WPM, a Discrete Wavelet Packet Transform (DWPT) based MCM scheme, is proposed as a solution to this problem [3]. WPM provides better spectral shaping than DFT-based MCM scheme. It offers much lower side lobes in transmitted signal, which reduces ICI and NBI [3]. It does not utilize CP for channel equalization and this conserves bandwidth. However, it requires a robust equalization technique to cancel ISI and ICI created by multipath wireless channel.

In literature, wavelet based MCM has been suggested to improve BER performance of transceiver in wireless communication [4] & [5]. However, no significant work is available on channel equalization in WPM systems. Jamin *et al.*, have given analysis of WPM for wireless communication, however, they have not proposed any technique with regards to channel equalization in wavelet based MCM systems [4]. Therefore, channel equalization of WPM system remains an open research area [4]. In this paper, we have proposed ZF and MMSE algorithms for training of a time-domain transversal equalizer. This performs equalization of multipath wireless channel in WPM systems. BER performance comparison is made with OFDM for multipath wireless channel. OFDM system utilizes a frequency-domain equalizer composed of a single tap per sub-carrier to compensate for channel distortion suffered by each sub-carrier.

II. Discrete Wavelet Packet Transform

The transmitted symbol $s(t)$ in a MCM scheme is constructed as the sum of amplitude modulated M waveforms $\varphi_m(t)$ as follows [4],

$$s(t) = \sum_{m=0}^{M-1} d_m \varphi_m(t) \quad (1)$$

where d_m is a constellation encoded *mth* data symbol modulating the *mth* waveform. The waveforms are mutually orthogonal [4],

$$\varphi_m(t) \otimes \varphi_n(t) = \delta(m-n) \quad (2)$$

$$\delta(i) = \begin{cases} 1 & \text{for} \quad i = 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\otimes$ represents a convolution operation.

OFDM, a DFT based MCM scheme, utilizes M complex exponentials $w(t)exp(2\pi mt/M)$ as the MCM basis functions $\varphi_m(t)$. These basis functions are limited in time-domain by the window function $w(t)$ which corresponds to a sinc-shaped waveform in the frequency domain, while, WPM utilizes wavelet packets as the MCM basis functions. Wavelet packets are obtained through the DWPT, which is implemented using digital filtering techniques [6]. Input signal $x[n]$ is decomposed by a low pass filter with impulse response g and a high-pass filter with impulse response f. However, since half the frequencies of the signal have now been removed at each filter output, half of the samples can be discarded according to the Nyquist's rule. Therefore, the low pass filter and a high-pass filter outputs are then down sampled by 2 to give approximation and detail coefficients, respectively, as shown in Fig. 1. The low pass and high pass filter output samples are given by [6],

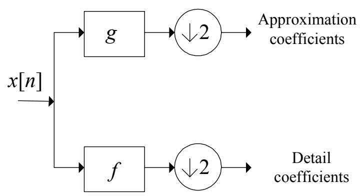

Fig. 1 Block diagram of filter decomposition

$$y_{low}[n] = \sum_{k=-\infty}^{k=+\infty} x[k]g[2n-k] \quad (3)$$

$$y_{high}[n] = \sum_{k=-\infty}^{k=+\infty} x[k]f[2n-k] \quad (4)$$

Both the detail and approximation coefficients are further decomposed by a series of high and low pass filters to give DWPT. DWPT gives the coefficients of wavelet packets. The original signal is reconstructed by performing the reverse operation of decomposition.

III. WPM System Model

System model for WPM is shown in Fig. 2. The input data stream is divided into parallel lower data sub-streams by a serial to parallel (S/P) converter. The data sub-streams are modulated by quadrature amplitude modulation (QAM). The QAM encoded variables are modulated onto the wavelet packets by inverse discrete wavelet packet transform (IDWPT). IDWPT takes the QAM encoded variables as input and gives WPM symbol to be transmitted, exactly as Inverse Discrete Fourier Transform (IDFT) in OFDM. The signal is convolved with the channel in the presence of additive white gaussian noise (AWGN). The multipath wireless channel distorts the signal and hence, induces ISI and ICI. In order to perform channel equalization, received signal is passed through a time-domain equalizer. While, DWPT is used for retrieving the data symbol transmitted exactly as DFT in OFDM. The original signal is recovered through QAM demodulation and parallel to serial conversion.

IV. Equalization Techniques for WPM

In order to cancel ISI and ICI caused by the distortion of the signal after it has passed through a multipath wireless channel, time-domain equalizer employing linear transversal filter is proposed. Coefficients of the equalizer are modified using MMSE and ZF algorithms. Channel impulse response is assumed to be known. MMSE and ZF equalization techniques are briefly discussed here.

A. *Zero-Forcing Equalization*

ZF-algorithm is based on the idea of canceling the channel effect completely by multiplying the received signal with the inverse of channel response [7] & [8]. In ZF-equalizer, the equalizer coefficients are chosen to force the samples of the

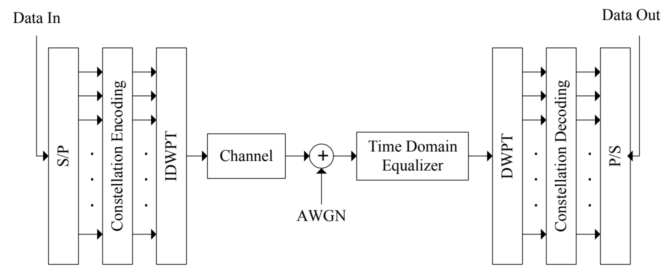

Fig. 2 Block diagram of WPM

combined channel and equalizer impulse response to zero at all sample points except the desired sample point [9]. The disadvantage associated with ZF-algorithm is that it excessively amplifies noise in the channel frequency nulls which degrades its BER performance [9]. If h is the received sequence of impulses and c are the equalizer weights then the equalizer output is given by [8],

$$z = hc \quad (5)$$

According to zero forcing criteria, equalizer weights are adjusted so as to cancel ISI [8],

$$z(k) = \begin{cases} 1 & \text{for} \quad k = 0 \\ 0 & \text{for} \quad k = \pm1, \pm2, ..., \pm N \end{cases} \quad (6)$$

The corresponding equalizer weights c are defined as,

$$c = h^{-1}z \quad (7)$$

This equation gives the equalizer coefficients c which forces the combined channel and equalizer impulse response to zero at all sample points except the desired sample point. For the channel impulse response of h=[h(-1) h(0) h(+1)], Eq. (5) can be rewritten as,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} h(0) & h(-1) & 0 \\ h(1) & h(0) & h(-1) \\ 0 & h(+1) & h(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} \quad (8)$$

The ZF-equalizer coefficients for different channel impulse responses are shown in Table I, where c represents a column vector and its elements are described in Table I.

B. *MMSE Equalization*

MMSE based equalizer provides a more robust solution as it minimizes mean-square error (MSE) of all ISI terms plus the noise power at the equalizer output [7], [8] & [9]. MSE is the expected value of squared difference between the desired equalizer output and actual equalizer output, hence it takes the channel noise effect into account. The equalizer output z for channel impulse response h is expressed as,

$$z = hc \quad (9)$$

In order to compute the optimum weights of MMSE equalizer, it is required to determine the cross-correlation and auto-correlation matrix. These are determined by modifying Eq. (9) as,

$$h^T z = h^T hc \quad (10)$$

$$R_{hz} = R_{hh}c \quad (11)$$

TABLE I
EQUALIZER COEFFICIENTS

Channel Impulse Response	ZF-Equalizer Coefficients	MMSE-Equalizer Coefficients
h_1=[0.407 1 0.407]	c_1=-0.6086, 1.4954, -0.6086	c_1=-0.4076, 1.4954, -0.4076
h_2=[0.507 1 0.507]	c_2=-1.0434, 2.0580, -1.0434	c_2=-0.3940, 1.1882, -0.3940
h_3=[0.707 1 0.707]	c_3=-2341.1, 3311.3, -2341.1	c_3=-0.0002, 0.5004, -0.0002

TABLE II
SIMULATION PARAMETERS

Parameter	**WPM**	**OFDM**
Data Rate	2-Mbps	2-Mbps
Sampling frequency	4- MHz	4-MHz
Modulation	4-ary QAM	4-ary QAM
Wavelet	db2	-
DWPT level	3-Level	-
CP length	-	3
No of sub-carriers	-	128

where R_{hz} is called cross-correlation matrix and R_{hh} is called auto-correlation matrix. Then the optimum weights for minimum MSE are [8],

$$c = R_{hh}^{-1} R_{hz} \tag{12}$$

For the channel impulse response of h=[h(-1) h(0) h(+1)], the channel matrix h is given by

$$h = \begin{bmatrix} h(-1) & 0 & 0 \\ h(0) & h(-1) & 0 \\ h(1) & h(0) & h(-1) \\ 0 & h(1) & h(0) \\ 0 & 0 & h(1) \end{bmatrix} \tag{13}$$

The MMSE equalizer coefficients for different channel impulse responses are shown in Table I, where c represents a column vector and its elements are described in Table I.

V. Simulation Results

In this section simulation results are presented which show the performance comparison of WPM and OFDM in terms of BER in a multipath wireless channel. WPM system utilizes ZF and MMSE equalizers for time-domain channel equalization. OFDM utilizes a frequency-domain equalizer composed of a single tap per sub-carrier to compensate for channel distortion suffered by each sub-carrier. Matlab is used as a simulation tool. Simulation parameters are specified in Table II. WPM transceiver shown in Fig. 1 is simulated with the assumption that the channel coefficients are known at the receiver.

Initially the channel coefficients represent the characteristics of a low pass filter that have frequency selective nature and no nulls in channel. Then coefficients were changed to have frequency nulls in channel frequency response as shown in Fig. 3, 5 & 7. For channel impulse response of h_1=[0.407 1 0.407], WPM utilizing MMSE-equalizer gives an

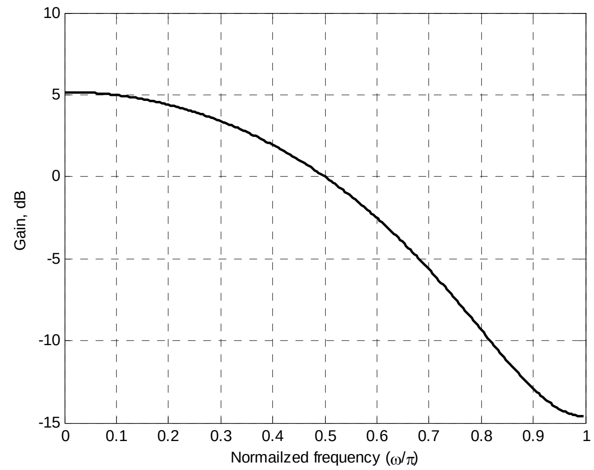

Fig. 3 Frequency response of h_1=[0.407 1 0.407]

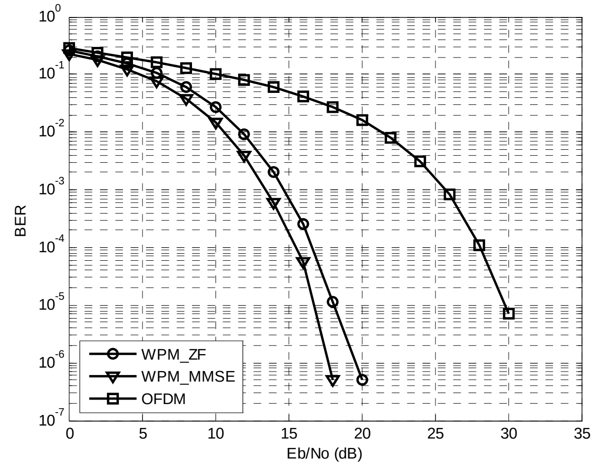

Fig. 4 BER performance comparison of WPM and OFDM for h_1=[0.407 1 0.407]

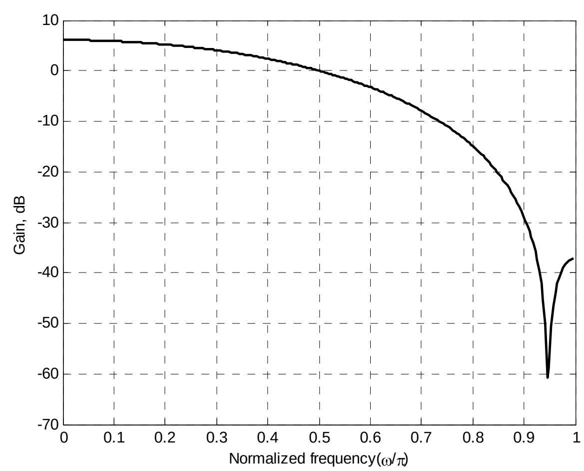

Fig. 5 Frequency response of h_2=[0.507 1 0.507]

improvement of 1.25 dBs in E_b/N_o over ZF-equalizer in WPM and 13 dBs in E_b/N_o over OFDM, at a BER of 10^{-5} as shown in Fig. 4. For the channel impulse response of h_2=[0.507 1 0.507],

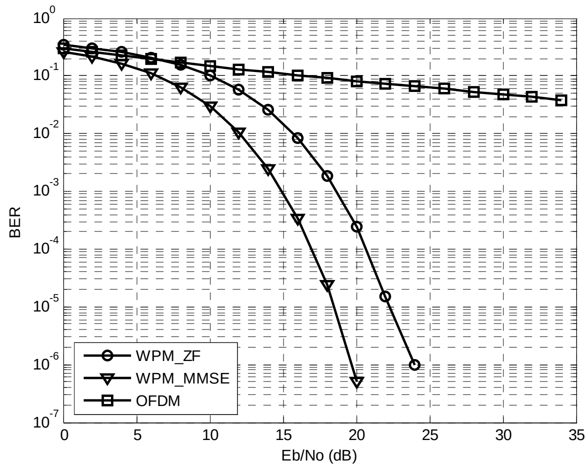

Fig. 6 BER performance comparison of WPM and OFDM for h_2=[0.507 1 0.507]

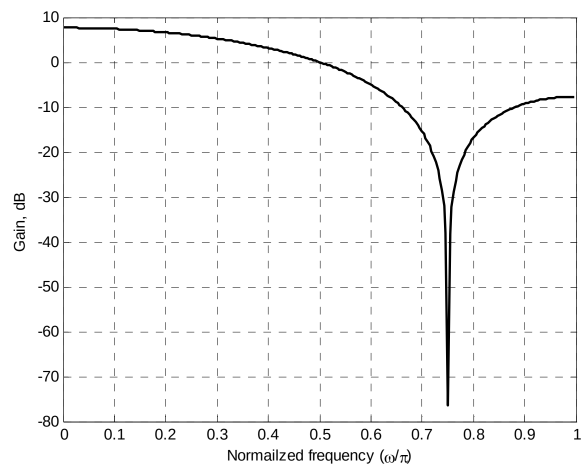

Fig. 7 Frequency response of h_3=[0.707 1 0.707]

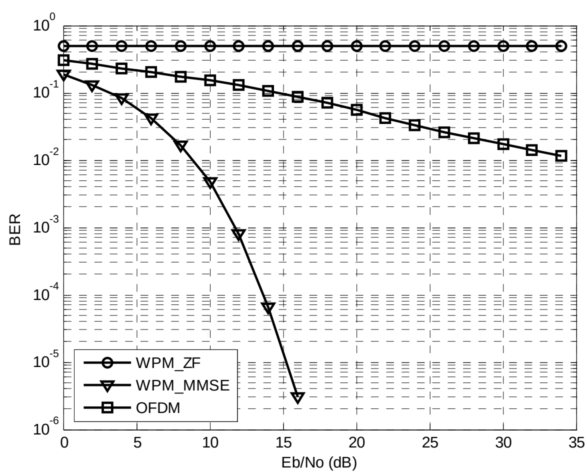

Fig. 8 BER performance comparison of WPM and OFDM for h_3=[0.707 1 0.707]

WPM utilizing MMSE-equalizer gives an improvement of 4 dBs in E_b/N_o over ZF-equalizer in WPM, at a BER of 10^{-6} while BER of OFDM degrades as shown in Fig. 6. When deeper frequency nulls are introduced in the channel, BER performance of OFDM and ZF-equalizer degrades severely due to the noise enhancement in frequency nulls as shown in Fig. 8.

Simulation results show that, WPM using MMSE equalization outperforms ZF-equalizer in WPM, as well as OFDM, for multipath wireless channel with AWGN. However**,** equalization of WPM in multipath channels is more complex than that of OFDM. As WPM utilizes an equalizer composed of three taps per sample for a three ray channel while OFDM utilizes an equalizer composed of one tap per sample.

VI. CONCLUSION

Performance comparison is made between WPM and OFDM for multipath wireless channel, with WPM utilizing MMSE and ZF-algorithms as channel equalization techniques. The simulation results show that WPM employing MMSE based channel equalization has a better BER performance as compared to OFDM in several wireless channels, but with higher complexity of equalization. For WPM, MMSE based equalization outperforms ZF-equalization in terms of BER as it takes the channel noise into account, while ZF-equalizer suffers from noise enhancement in channel frequency nulls.

Performance results indicate that WPM is a viable alternative to OFDM but at the cost of higher complexity of equalization. Although, OFDM offers a low complexity structure than WPM, however, the use of CP reduces its spectral efficiency and wastes transmit power. Moreover, OFDM suffers from noise enhancement in channel frequency nulls. As it employs a frequency–domain equalizer to cancel channel distortion of each sub-carrier which inverts the channel to generate complex coefficients for one-tap equalizer. Also, if the channel delay spread exceeds the CP length than it requires a time-domain equalization to shorten channel length. In that case, WPM and OFDM presents equalization complexity of the same order. Hence, WPM can emerge as a strong contender for MCM in multipath wireless channel.

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