DSP based Embedded Controller for Real Time Flight Control of an Autonomous Underactuated Aerial Robot

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***Abstract*- This paper outlines derivation of a nonlinear model of an underactuated quad rotor aerial robot, design a globally stable tracking nonlinear control for the quad rotor and validation of the designed controller using nonlinear simulation and real time flight tests. Model derivation comprises determining equations of motion of the quad rotor in three dimensions and seeking to approximate actuation forces through modeling of aerodynamic coefficients and electric motor dynamics.**

The derived model composed of translational and rotational subsystems is dynamically unstable. Affine nonlinear control strategy, using geometric methods of nonlinear control, is therefore implemented for control of the quad rotor. The stabilization of the nonlinear control strategy is evaluated using nonlinear simulation. Small, light weight sensors and a DSP, TMS320F2808, based embedded controller are used for the prototype quad rotor. The tracking performance of the nonlinear controller is evaluated using real time flight tests showing effectiveness of the proposed flight controller.

I. INTRODUCTION

In the last few years, there has been major interest in developing stabilizing algorithms for underactuated systems. This is due to the broad range of real life applications of underactuated systems in Robotics (e.g., mobile robots, exible-link robots, snake-type robots, walking robots), Aerospace Vehicles (e.g., aircraft, spacecraft, helicopters, and satellites), and Marine Vehicles (e.g., surface vessels and underwater vehicles). Quad rotor aerial robot is a highly nonlinear, multivariable, strongly coupled and underactuated system. The prototype quad rotor aerial robot designed for real time experimentation is as shown in figure (1).

Quad rotor aerial robots exhibit a number of important physical effects such as aerodynamic effects, inertial counter torques, gravity effect, gyroscopic effects and friction etc. Due to these effects it is difficult to design a real time control for aerial robots. Control of quad rotor aerial robot is accomplished by varying the speeds of four motors, each connected with a propeller. Quad rotor crafts naturally demand a sophisticated control system in order to allow for balanced flight. The dynamics of such a system demand constant adjustment of four motors simultaneously.

Figure 1: Prototype quad rotor aerial robot with protection jacket.

Quad rotor is dynamically elegant, inexpensive, and simple to design and construct. It is an Omni-directional vehicle, and has almost no constraints on its motion. It can be flown in tight spaces and does not require large safety distances to operate. These facts make it an ideal candidate for a versatile and user-friendly multi-agent test bed. However, the technical challenges for such systems are numerous. High thrust-to-weight ratios, endurance long enough to perform a meaningful mission, careful matching of batteries, electric motors, and rotors and weight optimization is very important for such systems.

Recent research work on quad rotor aerial robot includes dynamic modeling and configuration of a quad rotor aerial robot proposed by [1] and [2]. A control structure for quad rotor based on internal linearization was proposed by [3] while a quaternion based PD feedback control scheme for attitude stabilization of quad rotor aerial robot was developed by [4]. However the researchers treated underactuated quad rotor in a manner that the underactuated control problem is degenerated to a full actuation one. Augmenting commercially available quad rotor aerial robot with additional sensors has been used for control and experimentation by [5]. However using commercial quad rotor as a test bed for the control electronics has limitations on the payload the vehicle could carry and it could be expensive to repurchase the frame if a crash occurs.

Researchers also employed vision based stabilization and tracking control for quad rotor aerial robot, [6], [7]. However the vision system is not as fast as a gyro and it is not as reliable as other sensors due to sensitivity to changes in lighting conditions. A complete mathematical model of an underactuated quad rotor based on Newton Euler formalism, was derived by [8], using nonlinear control for aerial robot by transforming original system models into equivalent models of simpler form.

In this paper we consider flight control system design for a quad rotor based unmanned aerial robot with application to autonomous flight. In particular affine nonlinear controller, using Lie derivatives and Lie brackets, is designed and implemented for the attitude control of the quad rotor aerial robot. Affine nonlinear system possesses the feature that is nonlinear to state vector but linear to control variables, [9]. The derived dynamic model for the quad rotor aerial robot shows that it is differentially flat underactuated system since the inertia matrix of the dynamics is constant. For such class of underactuated mechanical systems, the dynamic feedback linearization based nonlinear method is the most feasible method.

II. Quad Rotor Dynamics

The main forces and moments acting on quad rotor are those produced by rotors. Two rotors in the system are counter rotating such that total torque of the system is balanced. The free body diagram and axes of quad rotor aerial robot is as shown in figure (2).

The basic motions of a quad rotor are generated by varying the rotor speeds of all four rotors, thereby changing the lift forces. Increasing or decreasing speed of the four motors together generates vertical motion. When motor pair (3, 1) is allowed to operate independently then the pitch angle θ (rotation about the y-axis) can be controlled along with the indirect control of motion along the x axis. Similarly when motor pair (2, 4) is allowed to operate independently then the roll angle ϕ (rotation about the x-axis) can be controlled along with the indirect control of motion along the y axis. Finally when motor pair (3, 1) is rotating clockwise and motor pair (2, 4) rotating counter-clockwise, the yaw angle ψ (rotation about the z-axis) can be controlled. The quad rotor aerial robot has now six degrees of freedom.

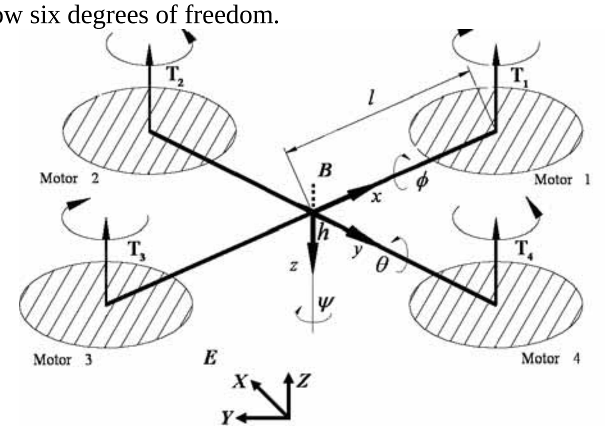

Figure 2: Axes of 6-DOF underactuated quad rotor aerial robot.

Where l represents distance of motor from pivot centre. h is vertical distance between propeller centre and CG of quad rotor. ϕ, θ and ψ represent Euler angles about x, y, z body axis respectively. T_n represents Thrust force produced by each rotor, for $n = [1,2,3,4]$. Earth fixed frame is represented by $E = \{X, Y, Z\}$ and body fixed frame is represented by $B = \{x, y, z\}$.

The aerodynamic forces and moments are derived using a combination of momentum and blade element theory, [10],[11]. The model developed for quad rotor assumes that the structure is supposed to be rigid and symmetrical. Quad rotor has four motors with propellers. A voltage applied to each motor results in a net torque being applied to the rotor shaft, Q_i, which results in a thrust, T_i. If the rotor disk is moving, there is a difference in relative velocity between the blade and air when moving through the forward and backward sweep, resulting in a net moment about the roll axis, R_i. Forward velocity also causes a drag force on the rotor that acts opposite to the direction of travel, D_i. Thrust and drag can be defined in terms of aerodynamic coefficients C_T and C_D as,

$$T = C_T \rho A r^2 \Omega^2 \quad (1)$$

$$D = C_D \rho A r^2 \Omega^2 \quad (2)$$

Where A is blade area, ρ is density of air, r is radius of the blade and Ω is angular velocity of propeller. Torque Q and rolling moment R, in terms of aerodynamic coefficients C_Q and C_R, are defined as,

$$Q = C_Q \rho A r^2 \Omega^2 r \quad (3)$$

$$R = C_R \rho A r^2 \Omega^2 r \quad (4)$$

Then total forces, f_{total} and moments τ_{total}, acting on quad rotor in body frame are given by,

$$f_{total} = \begin{bmatrix} -\frac{1}{2} A_c \rho \left(C_x \dot{x}|\dot{x}| + C_y \dot{y}|\dot{y}| + C_z \dot{z}|\dot{z}| \right) - \\ mgZ + \sum_{i=1}^{4} T_{zi} - \sum_{i=1}^{4} \left(D_{xi} + D_{yi}) \right) \end{bmatrix} \quad (5)$$

$$\tau_{total} = \begin{bmatrix} \sum_{i=1}^{4} (-1)^i Q_i + (-1)^{i+1} \sum_{i=1}^{4} \left(R_{xi} + R_{yi} \right) + h \sum_{i=1}^{4} \left(D_{xi} - D_{yi} \right) + \\ (T_4 - T_2) l x + (T_3 - T_1) l y + \left[\left((D_2 - D_4) + (D_3 - D_1) \right) l \right] z \end{bmatrix} \quad (6)$$

Where the first term in (5) represents the friction force on quad rotor body during horizontal motion with C_x, C_y, C_z representing longitudinal drag coefficients, A_c representing fuselage area and $\dot{x}, \dot{y}, \dot{z}$ representing speeds in x, y and z direction respectively, Z defines the vertical axis in inertial coordinates, m represents total mass of quad rotor while g

represents force due to gravity. In (6), $h(D_x), h(D_y)$ represent drag moment due to forward and sideward flight, R_x, R_y represent rolling moment due to forward and sideward flight while, $(D_{x2} - D_{x4})$ and $(D_{y3} - D_{y1})$ represent drag force unbalance in forward and sideward flight respectively.

Now consider quad rotor as a single rigid body with 6 DOF. Assuming that earth is flat and neglecting ground effect, the equations of motion for a rigid body subject to body force, $\boldsymbol{f}^b \in \Re^3$, and body moment, $\boldsymbol{\tau}^b \in \Re^3$, applied at the center of mass and expressed in Newton-Euler formalism, as in [12], are given by,

$$\begin{bmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{J} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}^b \\ \dot{\boldsymbol{\omega}}^b \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^b \times m\boldsymbol{v}^b \\ \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^b \\ \boldsymbol{\tau}^b \end{bmatrix} \tag{7}$$

Where $\mathbf{v}^b \in \Re^3$ is body velocity vector, $\boldsymbol{\omega}^b \in \Re^3$ is body angular velocity vector, $\mathbf{I} \in \Re^{3\times3}$ is an identity matrix, and $\mathbf{J} \in \Re^{3\times3}$ is an inertial matrix.

A. Rotational Dynamics

Assuming that the inertia tensor is diagonal (symmetric design of quad rotor), the moment equations governing the quad rotor are given by,

$$\boldsymbol{\tau}^b = \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b + \boldsymbol{\tau}_{total} \tag{8}$$

From (8) and (6), rotational dynamics of quad rotor in body axis are given by,

$$J_x\ddot{\phi} = \dot{\theta}\dot{\psi}(J_y - J_z) + l(T_4 - T_2) + \sum_{i=1}^{4}(-1)^{i+1}R_{xi} - h\sum_{i=1}^{4}D_{yi} \tag{9}$$

$$J_y\ddot{\theta} = \dot{\phi}\dot{\psi}(J_z - J_x) + l(T_3 - T_1) + \sum_{i=1}^{4}(-1)^{i+1}R_{yi} + h\sum_{i=1}^{4}D_{xi} \tag{10}$$

$$J_z\ddot{\psi} = \dot{\phi}\dot{\theta}(J_x - J_y) + \sum_{i=1}^{4}(-1)^{i}Q_i + \left((D_{x2} - D_{x4}) + (D_{y3} - D_{y1})\right)l \tag{11}$$

B. Translational Dynamics

Effect of body moments on the translational dynamics is neglected. Then from (7) and (5) the translational dynamics governing quad rotor are given by,

$$m\ddot{X} = \left[(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\sum_{i=1}^{4}T_i - \sum_{i=1}^{4}D_{xi} - \frac{1}{2}C_x A_c \rho\dot{x}|\dot{x}|\right] \tag{12}$$

$$m\ddot{Y} = \left[(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\sum_{i=1}^{4}T_i - \sum_{i=1}^{4}D_{yi} - \frac{1}{2}C_y A_c \rho\dot{y}|\dot{y}|\right] \tag{13}$$

$$m\ddot{Z} = -mg + (\cos\phi\cos\theta)\sum_{i=1}^{4}T_i - \frac{1}{2}C_z A_c \rho\dot{z}|\dot{z}| \tag{14}$$

III. AERODYNAMIC COEFFICIENTS

A relationship between the thrust produced and the velocity communicated to the air can be obtained by the application of Newtonian mechanics to the overall process, as in [10]. The standard blade element notation is as shown in figure 3.

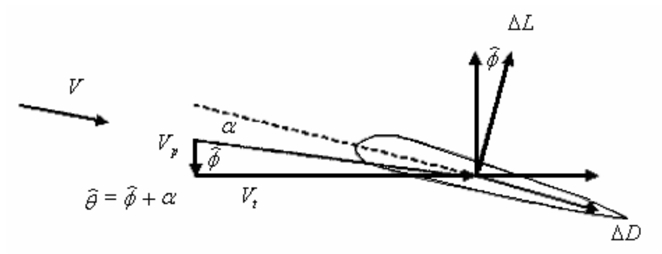

Figure 3: Blade element notation.

Where $\hat{\theta}$ is the blade angle w.r.t. the rotor shaft, α is the resultant angle of attack, V, V_p, V_t, the air velocity and components relative to the blade element, $\hat{\phi}$ is the velocity of airflow angle w.r.t. the rotor shaft ($\hat{\phi} = V_p / V_t$ for small angles), and $\hat{\psi}$ is the blade heading or azimuth which is zero at the point that is opposite to the vehicle motion.

Thus the thrust due to resultant of the vertical forces acting on all the blade elements is,

$$T = N\rho a\bar{c}(\Omega R)^2 R\left(\left(\frac{1}{6} + \frac{1}{4}\mu^2\right)\hat{\theta} - \frac{1}{4}\lambda\right) \tag{15}$$

Where N is the number of blades and $\bar{c}$ represents the average rotor chord, λ is the inflow ratio, μ is rotor advance ratio and a defined as lift slope.

Thus the model for C_T can be obtained as,

$$\frac{C_T}{\sigma a} = \left(\left(\frac{1}{6} + \frac{1}{4}\mu^2\right)\hat{\theta} - \frac{1}{4}\lambda\right) \tag{16}$$

Where σ is rotor solidity ratio. Similarly C_D, C_Q, C_R model are given by,

$$\frac{C_D}{\sigma a} = \left(\frac{1}{4a}\mu\overline{C}_d + \frac{1}{4}\lambda\mu\hat{\theta}\right) \tag{17}$$

Where $\overline{C}_d$ is defined as drag coefficient at 70% radial station.

$$\frac{C_Q}{\sigma a} = \frac{1}{8a}(1 + \mu^2)\overline{C}_d + \lambda\left(\frac{1}{6}\hat{\theta} - \frac{1}{4}\lambda\right) \tag{18}$$

$$\frac{C_R}{\sigma a} = \mu\left(\frac{1}{6}\hat{\theta} - \frac{1}{8}\lambda\right) \tag{19}$$

IV. ENGINE MODEL

On electrical side of DC motor, a current I_a flows through armature according to drive voltage V_a, motor's inductance L_m, resistance R_m and back emf voltage V_{emf}, then,

$$V_a - V_{emf} = L_m\frac{dI_a}{dt} + R_m I_a \tag{20}$$

Motor converts electrical armature current into a mechanical torque applied to shaft, $T_m = K_{tm}I_a$. The applied torque produces angular velocity ω_m according to inertia J_m and motor load T_l as,

$$T_m = J_m \frac{d\omega_m}{dt} + T_l \tag{21}$$

Defining $V_{emf} = K_e \omega_m$, neglecting inductance of the small motor and introducing propeller and gearbox models, then from (20) and (21) we have,

$$\dot{\omega}_m = -\frac{K_{tm} K_e}{R_m J_m}\omega_m - \frac{d}{\eta r_g^3 J_m}\omega_m^2 + \frac{K_{tm}}{R_m J_m} V_a \tag{22}$$

Where η is gear box efficiency, d is drag factor and r_g is gear box reduction ratio.

V. Control Strategy

The derived model is simplified for control design in order to comply with the real time constraints of the embedded control loop. Drag forces and rolling moments are neglected and thrust and drag coefficients are supposed to be constant. Thus the inputs to quad rotor aerial robot, i.e. vertical force input u_1, roll moment input u_2, pitch moment input u_3 and yaw moment input u_4, based on these assumptions are defined as,

$$\left.\begin{aligned} u_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ u_2 &= b(\Omega_4^2 - \Omega_2^2) \\ u_3 &= b(\Omega_1^2 - \Omega_3^2) \\ u_4 &= d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{aligned}\right\} \tag{23}$$

Where b and d are thrust and drag factors respectively.

A. Rotational Control

Affine nonlinear control design is implemented for the quad rotor aerial robot. The focus is on the attitude controller which is the main controller of quad rotor aerial robot. Affine nonlinear control system is of the form,

$$\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g_1}(\boldsymbol{x})\boldsymbol{u}_1 + \ldots + \boldsymbol{g_m}(\boldsymbol{x})\boldsymbol{u_m} \tag{24}$$

Where $\mathbf{x} \in \aleph \subset \Re^n$, with $\aleph$ having the origin as an interior point, $\boldsymbol{u} \in \Re^m$ and $\boldsymbol{g_1}(\boldsymbol{x}) + \ldots + \boldsymbol{g_m}(\boldsymbol{x})$ are column vectors of $\mathbf{g}(\mathbf{x})$. It is assumed that $\mathbf{f}$ and $\boldsymbol{g_1} + \ldots + \boldsymbol{g_m}$ are sufficiently smooth and without loss of generality it is further assumed that the origin is an equilibrium state, i.e., $\boldsymbol{f}(0) = 0$. For quad rotor aerial robot function vectors $\boldsymbol{f(x)} \in \Re^6$, $\mathbf{g_i} \in \Re^6$ and output vector $\mathbf{y}$ are given by,

$$\boldsymbol{f(x)} = \begin{bmatrix} x_2 & (x_4 x_6)\left(\frac{J_y - J_z}{J_x}\right) & x_4 & (x_2 x_6)\left(\frac{J_z - J_x}{J_y}\right) & x_6 & (x_2 x_4)\left(\frac{J_x - J_y}{J_z}\right) \end{bmatrix}^T \tag{25}$$

$$\boldsymbol{g}_1\boldsymbol{(x)} = \begin{bmatrix} 0 & \frac{l}{J_x} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{26}$$

$$\boldsymbol{g}_2\boldsymbol{(x)} = \begin{bmatrix} 0 & 0 & 0 & \frac{l}{J_y} & 0 & 0 \end{bmatrix}^T \tag{27}$$

$$\boldsymbol{g}_3\boldsymbol{(x)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{J_z} \end{bmatrix}^T \tag{28}$$

$$\boldsymbol{y} = \begin{bmatrix} x_1 & x_3 & x_5 \end{bmatrix}^T \tag{29}$$

The states are defined as, $\boldsymbol{x} = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$. A nonlinear control system (24) is feedback linearizable if there exists a coordinate transformation that transforms the system to a companion form and a nonlinear feedback control law that cancels the non linear dynamics, thus reducing the original system to a linear controllable system.

From [9], a $k-$dimensional vector field $\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_k}(\boldsymbol{x})$ defined on the open subset *Ꝺo* of $\aleph$, is a mapping that assigns a $k-$dimensional vector to each point $\mathbf{x}$ of *Ꝺo*.

Definition 1: A $k-$dimensional distribution $\Delta(.)$ on *Ꝺo* is a map that assigns to each $x \in$ *Ꝺo*, a $k-$dimensional subspace $\Delta(\mathbf{x})$ of $\aleph$, such that there exists a $k-$dimensional vector field $\mathbf{f}_1(\mathbf{x}), \mathbf{f}_2(\mathbf{x}), \ldots, \mathbf{f}_k(\mathbf{x})$ satisfying, $\{\mathbf{f}_1(\mathbf{x}), \mathbf{f}_2(\mathbf{x}), \ldots, \mathbf{f}_k(\mathbf{x})\}$ is a linearly independent set, $\mathbf{x} \in$ *Ꝺo*. $\Delta(\mathbf{x}) = span\{\mathbf{f_1}(\mathbf{x}), \mathbf{f_2}(\mathbf{x}), \ldots, \mathbf{f}_k(\mathbf{x})\}, \mathbf{x} \in$ *Ꝺo*.

Let $[\mathbf{f_i}, \mathbf{f_j}]$ denote the Lie bracket,

$$[\mathbf{f_i}, \mathbf{f_j}] = ad_{f_i}\mathbf{f_j} \stackrel{\Delta}{=} \frac{\partial \mathbf{f_j}}{\partial \mathbf{x}}\mathbf{f_i} - \frac{\partial \mathbf{f_i}}{\partial \mathbf{x}}\mathbf{f_j} \tag{30}$$

Higher order and zero order Lie brackets are defined as,

$$ad_{f_i}^k \mathbf{f_j} = ad_{f_i} ad_{f_i}^{k-1}\mathbf{f_j} \qquad \mathrm{ad}_{\mathrm{f_i}}^0 \mathbf{f_j} = \mathbf{f_j} \tag{31}$$

Definition 2: The distribution $\Delta(\mathbf{x})$ is involutive if $[\mathbf{f_i}, \mathbf{f_j}] \in \Delta(\mathbf{x}), \mathbf{f_i}, \mathbf{f_j} \in \Delta(\mathbf{x})$.

For the controlled system (24) define the distributions,

$$\Delta_0(\mathbf{x}) \stackrel{\Delta}{=} span\{\mathbf{g_1}(\mathbf{x}), \mathbf{g_2}(\mathbf{x}), \ldots, \mathbf{g}_m(\mathbf{x})\} \tag{32}$$

$$\Delta_1(\boldsymbol{x}) \stackrel{\Delta}{=} span\left\{\begin{matrix} \boldsymbol{g_1}(\boldsymbol{x}), \boldsymbol{g_2}(\boldsymbol{x}), \ldots, \boldsymbol{g}_m(\boldsymbol{x}), \\ ad_f \boldsymbol{g_1}(\boldsymbol{x}), ad_f \boldsymbol{g_2}(\boldsymbol{x}), \ldots, ad_f \boldsymbol{g_m}(\boldsymbol{x}) \end{matrix}\right\} \tag{33}$$

$$\Delta_i(\mathbf{x}) \stackrel{\Delta}{=} span\{ad_f^k \mathbf{g}_j(\mathbf{x}), k = 0,1,\ldots,i,\ j = 1,2,\ldots,m\} \tag{34}$$

The following theorem is from [14].

Theorem 1: Suppose rank $\mathbf{g}(0) = m$. Then (24) is feedback linearizable if and only if,

1. For each $0 \le i \le n-1$, the distribution $\Delta_i(\mathbf{x})$ has constant dimension in a neighborhood of the origin.

2. The distribution $\Delta_{n-1}(\boldsymbol{x})$ has constant dimension n in a neighborhood of the origin.
3. For each $0 \le i \le n-2$ the dimension $\Delta_i(\boldsymbol{x})$ is involutive in a neighborhood of the origin.

To check the distribution of $\Delta(\mathbf{x})$ for involutivity, the matrices $\Delta_o(\mathbf{x})$ and $\Delta_1(\mathbf{x})$ are obtained as given by (32) and (33). The matrices, $\Delta_o(\boldsymbol{x})$ and $\Delta_1(\boldsymbol{x})$, have equal rank. Thus the new vector field obtained by Lie bracket operation of two arbitrary vector fields from the original k vector fields (i.e. evaluating the derivative of one vector field along the other) is not linearly independent but linearly dependent with the previous k vector fields. In other words, the new vector field is still in the original spanned space of k vector fields and does not form a new direction. Thus the augmented matrix has the rank that still equals rank of the original matrix. Therefore $\Delta_o(\boldsymbol{x})$ is involutive. Moreover all the conditions of theorem 1 are satisfied and the affine nonlinear system is feedback linearizable.

Next the relative degree of rotational subsystem is investigated and it is found out that the Lie derivative of the function $L_f^k h(\mathbf{x})$ along $\mathbf{g}$ equals zero in a neighborhood of $\Delta_o(\boldsymbol{x})$ and the Lie derivative of the function $L_f^{k-1} h(\boldsymbol{x})$ along vector field $\mathbf{g}(\mathbf{x})$ is not equal to zero. Thus the relative degree of the rotational subsystem equals the order of the system. Thus there are no internal dynamics and both state regulation and output tracking can be achieved easily.

Next the diffeomorphism $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6) = \varsigma_F(\boldsymbol{x})$ that transforms (24) to a linear controllable system in Brunovsky form can be obtained by solving the partial differential equations,

$$L_{g_1} h_1(\mathbf{x}) = 0, L_{g_2} h_2(\mathbf{x}) = 0, L_{g_3} h_3(\mathbf{x}) = 0 .$$

Since $\Delta_o(\mathbf{x})$ is involutive, the Frobenius theorem guarantees that solution exists.

$\boldsymbol{\zeta} = \varsigma_F(\mathbf{x}) = (x_1, x_2, x_3, x_4, x_5, x_6)$ and the diffeomorphism $\varsigma_F(\mathbf{x}): Đ_o \rightarrow \Re^6$ is defined globally in *Đ*.

Then (24) can be transformed to Brunovsky form by defining, $\mathbf{w} = (w_1 \quad w_2 \quad w_3)^T$, where,

$$w_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_{g_j} L_f^{r_i-1} h_i u_j \tag{35}$$

In our case, $i = (1,2,3)$ and $\mathrm{m} = (1,2,3)$. A routine calculation shows that,

$$w_1 = \left[x_4 x_6 \left(\frac{J_y - J_z}{J_x} \right) + \frac{l}{J_x} u_1 \right] \tag{36}$$

$$w_2 = \left[x_2 x_6 \left(\frac{J_z - J_x}{J_y} \right) + \frac{l}{J_y} u_2 \right] \tag{37}$$

$$w_3 = \left[x_2 x_4 \left(\frac{J_x - J_y}{J_z} \right) + \frac{1}{J_z} u_3 \right] \tag{38}$$

From (36), (37) and (38),

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \mathbf{E}^{-1} \left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} - \begin{bmatrix} x_4 x_6 \left(\frac{J_y - J_z}{J_x} \right) \\ x_2 x_6 \left(\frac{J_z - J_x}{J_y} \right) \\ x_2 x_4 \left(\frac{J_x - J_y}{J_z} \right) \end{bmatrix} \right) \tag{39}$$

Where $\mathbf{E}$ is a decoupling square matrix defined obviously.

Thus utilizing the diffeomorphism $\boldsymbol{\zeta} = \varsigma_F(\boldsymbol{x})$ and substituting (39) into (24) yields the Brunovsky form. Any linear control law that stabilizes (39) can be used to stabilize the original system (24) by performing the inverse diffeomorphism $\boldsymbol{x} = \varsigma_F^{-1}(\boldsymbol{\zeta})$ along with the control transformation. The control law (39) with a linear controller asymptotically stabilizes the attitude angles of the quad rotor aerial robot. Moreover, the stabilization is global on *Đ*, since $\boldsymbol{x} = \varsigma_F^{-1}(\boldsymbol{\zeta})$ is a diffeomorphism on *Đ* and the control law (39) is well defined on *Đ*.

B. *Altitude Control*

Consider the altitude subsystem of the quad rotor aerial robot given by,

$$\dot{Z} = -g + \frac{(\cos\phi \cos\theta)}{m} u_1 \tag{40}$$

The altitude subsystem of the quad rotor can be linearized by selecting input u_1 as,

$$u_1 = \frac{mg}{\cos\phi\cos\theta} + \frac{v}{\cos\phi\cos\theta} \tag{41}$$

The necessary condition for (41) is $\cos\phi\cos\theta \neq 0$. v can be a PD controller, given by,

$$v = K_d \dot{z} + K_p (Z - Z_d) \tag{42}$$

Where K_p and K_d are proportional and derivative positive gains, respectively, and Z_d is the desired altitude.

C. *Position Control*

Let $\dot{x}_d$ and $\dot{y}_d$ be desired speeds in x and y direction respectively. Then error in desired and actual speeds is,

$$e_x = \dot{x}_d - \dot{x} \tag{43}$$

$$e_y = \dot{y}_d - \dot{y} \tag{44}$$

From the position subsystem, the desired roll and pitch angles, in terms of the error between actual and desired speed, are given by,

$$\phi_d = \sin^{-1}(u_{ex} \sin\psi - u_{ey} \cos\psi) \quad (45)$$

$$\theta_d = \sin^{-1}\left[\frac{u_{ex}}{\cos\phi\cos\psi} - \frac{\sin\phi\sin\psi}{\cos\phi\cos\psi}\right] \quad (46)$$

Where,

$$u_{ex} = K_x e_x \frac{m}{u_1}, \quad u_{ey} = K_y e_y \frac{m}{u_1} \quad (47)$$

Where K_x, K_y are positive constants and u_1 is vertical force input from altitude control.

VI. Simulation Results

Let us simulate the closed loop system with nonlinear control algorithm. The physical parameters of quad rotor aerial robot for nonlinear simulation are, l = 0.3020 m, J_x = 0.0154 kg m^2, J_y = 0.0150 kg m^2, J_z =0.0309 kg m^2, m=0.6500 kg. The initial conditions used are $\phi = \theta = \psi = 30°$ and $Z = 1$ meters. The reference input to the controller are, $\dot{x}_d = \dot{y}_d = 0$, $Z_d = 1$ and $\psi_d = 0$.

Figure (4) shows the response of the nonlinear controller to stabilize the quad rotor at hover. Simulation results, shown in figure (4), are performed with a model which includes actuators' dynamics. Although very hard initial conditions are used yet it can be seen that the controller effectively controls the roll, pitch and yaw angles of quad rotor in less than 6 seconds.

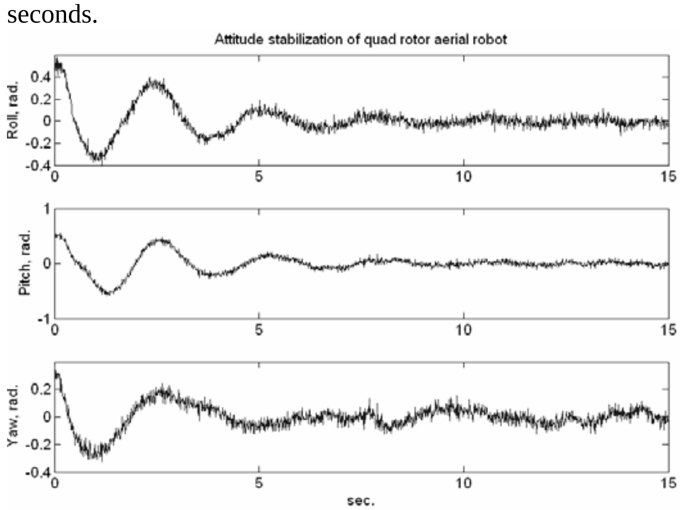

Figure 4: Attitude response of the quad rotor aerial robot.

The altitude and position response of the quad rotor aerial robot are shown in figure (5). Results from figure (5) indicate that the position controller effectively makes the attitude controller keep the quad rotor aerial robot at a given point.

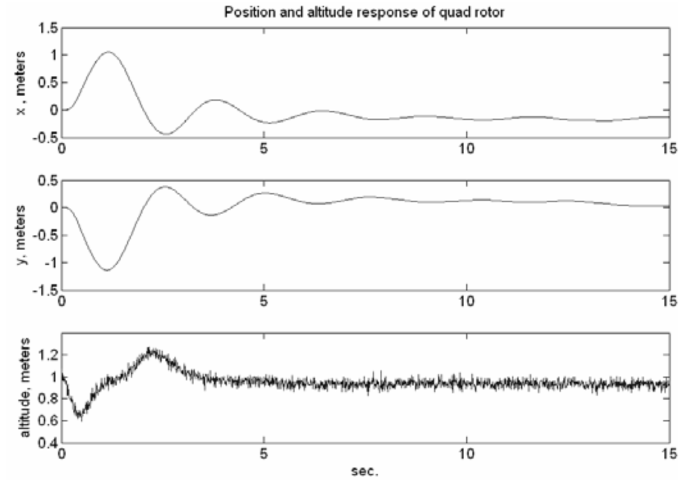

Figure 5: Altitude and position rate response of quad rotor aerial robot.

Figure (6) shows the rotor speed response of a quad rotor. The two pair of propellers (1, 3) and (2, 4) rotating in opposite direction, effectively cancels the moments of the quad rotor aerial robot.

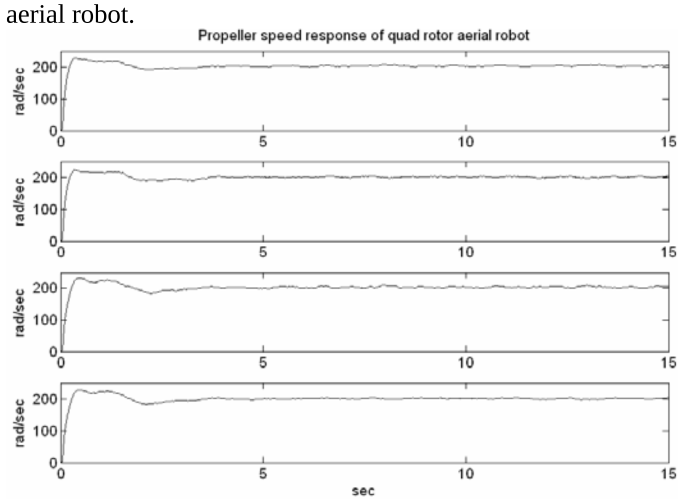

Figure 6: Rotor speed response of the quad rotor aerial robot.

VII. Experimental Results

The prototype quad rotor aerial robot is designed and developed to be a fully self contained unit, featuring onboard power, wireless communication, and onboard processing. After analyzing the structure characteristics and control requirements of the quad rotor, an embedded flight controller for the aerial robot, based on DSP TMS320F2808, is designed Posture control experiments are carried out using the controller, and the results show that the controller can accomplish expected tasks such as wireless communication, high resolution posture acquisition and closed-loop control effectively. The quad rotor is instrumented with a six degrees-of-freedom inertial measurement unit (IMU) and Sonar (SRF05). The IMU measures accelerations and rotation rates at a high update rate. The electronic system is designed to read the sensor data, control the actuators, and send telemetry data back to the base station. The DSP processor acts as the central processing hub, so it communicates with all sensors and the motor control units. Given the inertial data from the IMU and altitude data from Sonar, the main processor computes desired propeller speeds. These are sent to each motor control unit and the quad rotor is stabilized. Figure (7) shows the attitude tracking performance of the nonlinear controller.

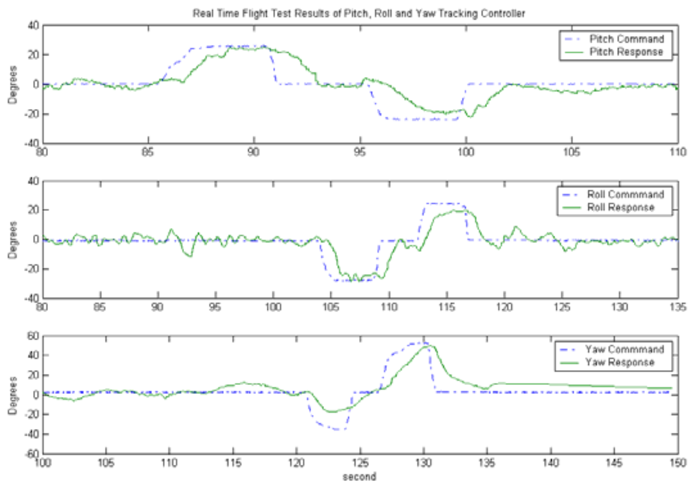

Figure 7: Tracking response of the quad rotor aerial robot.

Figure (8) shows the PWM of the front, back, left and right motors.

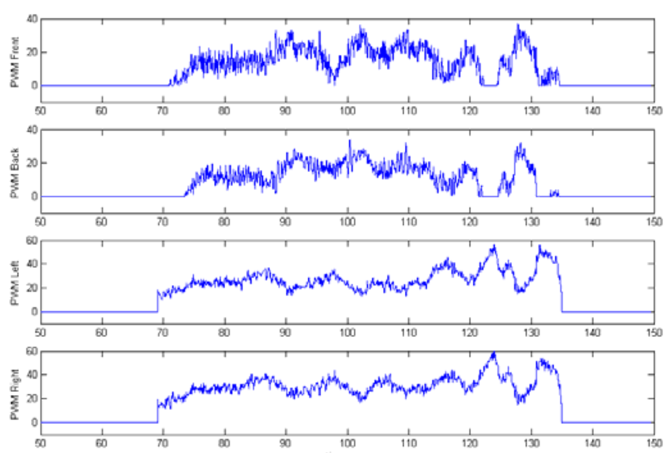

Figure 8: PWM of all the four motors.

VIII. CONCLUSION

We presented a stabilization nonlinear control method for quad rotor aerial robot. The modeling of the quad rotor is based on Newton-Euler formalism. From the nonlinear simulation results it can be seen that the nonlinear controller effectively controls the quad rotor aerial robot. The linearization accomplished for rotational subsystem is an "exact feedback linearization" as opposed to the conventional "Jacobian linearization" (Taylor-series expansion). Feedback linearization is based on the idea of transforming nonlinear dynamics into a linear form by using state feedback with input state linearization. The rotational subsystem of the quad rotor aerial robot is decoupled under proper state transformation, using Lie derivative and Lie brackets i.e. geometric methods of nonlinear control. The global stability of the rotational subsystem is ensured by the fact that relative degree equals the system order and hence there are no internal dynamics. The stabilization ability of the nonlinear controller is examined through nonlinear simulation and results indicate effectiveness of the proposed control strategy for the quad rotor aerial robot near quasi stationary flight. The tracking ability of the controller is verified by real time flight test results.

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