A Low Complexity Optimum Multi-user Receiver for Maximizing the BER Performance for DS-CDMA Systems

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Abstract- **A new transformation matrix technique for reducing the complexity of a multiuser receiver for DS-CDMA system is presented. The reduction in complexity of multiuser receiver would result in better bit error rate (BER) performance. The reduction in error rate would allow us to maximize the data throughput of a communication network by minimizing the packet loss. Our simulation results demonstrate that the proposed technique successfully reduces the computational complexity of an optimal multiuser receiver for the DS-CDMA systems. The complexity of the proposed technique is not polynomial in the number of users, but it still gives comparatively reduced complexity that can be used to achieve optimum performance in terms of a reduced BER and increased network data throughput.**

Keywords- DS-CDMA, bit error rate, data throughput, multiuser communications, packet loss

I. INTRODUCTION

Recovery of transmitted signals at the receiver is achieved by demodulation techniques [5]. A widely used technique called the single user matched filter regards the Code division multiple access (CDMA) channel as a single user channel and considers the detection problem of each user individually. In the absence of user interference, the single user matched filter is optimal in the sense of minimizing the bit error rate. However, this is no longer true in CDMA systems where MAI is present. To overcome these disadvantages of a single user matched filter, multiuser detectors have been developed [2, 3].

Multiuser direct-sequence (DS) CDMA has received wide attention in the field of wireless communications [1]. With the emergence of multiple access techniques, there has been an increase in the interest in performing simultaneous estimation and detection at the same time over all users [3].

Multiuser detection is an important technology in wireless CDMA systems for improving both data rate and the network capacity. Verdu's [2] proposed and analyzed the optimum multiuser detector and the maximum likelihood sequence detector, which, unfortunately, is too complex for practical implementation, since its complexity grows exponentially as the function of the number of users. Consequently, Verdu's work inspired researchers to find suboptimal multiusers detectors, which could achieve a close to optimal performance with reasonable asymptotic computational complexity. Suboptimal receivers also simultaneously detect all user signals. However, instead of maximum likelihood (ML) detection, they perform a set of linear transformations on the outputs of a matched filter that significantly enhance the noise component. Multiuser detection, which is the demodulation of users in the presence of MAI, is therefore essential for the efficient operation of a DS-CDMA system.

In this paper, we work on the Verdu's original algorithm for optimum multiuser receiver that has the main advantage of reaching *minimum mean square error* (MMSE) performance. We demonstrate that the proposed system can reduce the asymptotic computational complexity of multiuser receivers by using transformation matrix technique. By using the proposed system which provides a novel transformation matrix technique for complexity-reduction, the computational complexity of multiuser receivers can be reduced by several orders of magnitude. Thus this reduction in complexity would likely to give us a considerable improvement in the performance of multiuser receivers. The performance measure adopted in this paper is the achievable bit rate for a fixed probability of error (10^{-7}) and the consistent values of SNR.

The rest of the paper is organized as follows. Section II presents the state of the art research. Section III presents both the proposed transformation matrix technique and the original ML algorithm with their corresponding computational complexities. Section IV presents the BER performance of the proposed technique. Finally, Section V concludes the paper.

II. RELATED WORK

Multiuser receivers can be categorized in the following two forms: optimal maximum likelihood sequence estimation (MLSE) receivers and suboptimal linear and nonlinear receivers. In case of synchronous CDMA system, two main criteria are employed, namely the zero-forcing (ZF) and the MMSE. Both mechanisms can implement in two possible ways [8]. In the first option, both of them can be implemented to deal simultaneously with ISI and MAI where as in the second option, they deal only with ISI [10]. Even though this tedious computation is performed only once, a reduced complexity inversion algorithm may increase the overall performance [1]. Two well known classes of CDMA adaptive multiuser detection are *trained* and *blind* detectors. The trained detector is a robust adaptive detector that does not require the knowledge of spreading code of the desired user [10]. Results have shown [4] that these receivers are robust. The blind detector is a powerful adaptive detector that does not require any preliminary information about the data sequence [9].

Non-linear multiuser receiver involves the estimation and reconstruction of MAI [8] seen by each user with the objective of canceling it from the received signal. The two well known implementations of this mechanism are SIC and PIC. In interference cancellation, MAI is first estimated and then subtracted from the received signal [5, 7]. On the other hand, linear multiusers receivers apply a linear transformation to an observation vector, which serves as soft decision for the transmitted data. Recently, Ottosson and Agrell [6] proposed a new ML receiver that uses the neighbor descent (ND) algorithm. They implemented a linear iterative approach using the ND algorithm to locate the region where the actual observations belong. The linearity of their iterative approach increases noise components at the receiving end. Due to the enhancement in the noise components, the SNR and BER of ND algorithm is more affected by the MAI. Table 1, reported from [8], highlights the assumed knowledge for the computational complexity of a CDMA based multiuser receiver. Table I shows that different receivers distinguish themselves with respect to the requirement of the desired knowledge as well as the implementation complexity.

III. Proposed Transformation Matrix Technique

This section presents both the original ML algorithm and the proposed transformation matrix technique along with their corresponding computational complexities. Before presenting the proposed system, it is worth mentioning some of our key assumptions.

A. *Proposed Model and Key Assumptions*

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. Our transformation matrix technique utilizes the complex properties of the existing inverse matrix algorithms to construct the transformation matrices and to determine the location of the transformation points that may occur in any coordinate of the constellation diagram. The individual transformation points can be used to determine the average computational complexity.

The system may consist of K users. User k can transmit a signal at any given time with the power of W_k. With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, i.e., $b_k \in \{\pm 1\}$. The cross correlation can be reduced by neglecting the variable delay spreads, since these delays are relatively small as compared to the symbol transmission time. In order to detect signals from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator. It should be noted that we extract the signal using the match filter followed by a Viterbi algorithm.

B. *Computational Complexity of an ML Receiver*

The optimum multiuser receiver exists and permits to relax the constraints of choosing the spreading sequences with good correlation properties at a cost of increased receiver complexity. Fig. 1 shows the block diagram of an optimum receiver that uses a bank of matched filters and a maximum likelihood Viterbi decision algorithm for signal detection. In order to detect signal from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator.

When receiver wants to detect the signal from user-1, it first demodulates the received signal to obtain the base-band signal. The base-band signal multiplies with user-1's unique signature waveform, $C_1(t)$. The resulting signal, $r_1(t)$, is applied to the input of the matched filter. The matched filter integrates the resulting signal $\{r_1(t)\}$ over each symbol period T, and the output is read into the decoder at the end of each integration cycle. The outputs of the matched filter and the Verdu's algorithm can be represented by $y_k(m)$ and $b_k(m)$, respectively where m is the sampling interval. We also assume that the first timing offset τ_1 is almost zero and $\tau_2 < T$. The same procedure applies to other users. The outputs of the matched filter for the first two users at the m^{th} sampling interval can be expressed as follows:

$$y_1(m) = \frac{1}{T}\left\{ \int_{(m)T}^{(m+1)T} r_1(t) C_1(t) dt \right\} \quad (1)$$

TABLE I
COMPLEXITY REQUIREMENTS OF SOME DETECTION ALGORITHMS FOR CDMA SYSTEMS

Receivers	Signature of Desired User	Signature of Interference	Timing of Desired User	Timing of Interferers	Relative Amplitude	Training Sequence
Conventional and Rake	YES	NO	YES	NO	YES	NO
Linear ZF	YES	YES	YES	YES	NO	NO
Linear MMSE	YES	YES	YES	YES	YES	NO
SIC and PIC	YES	YES	YES	YES	YES	NO
Trained Adaptive MMSE	NO	NO	YES	NO	NO	YES
Blind Adaptive MMSE	YES	NO	YES	NO	NO	NO

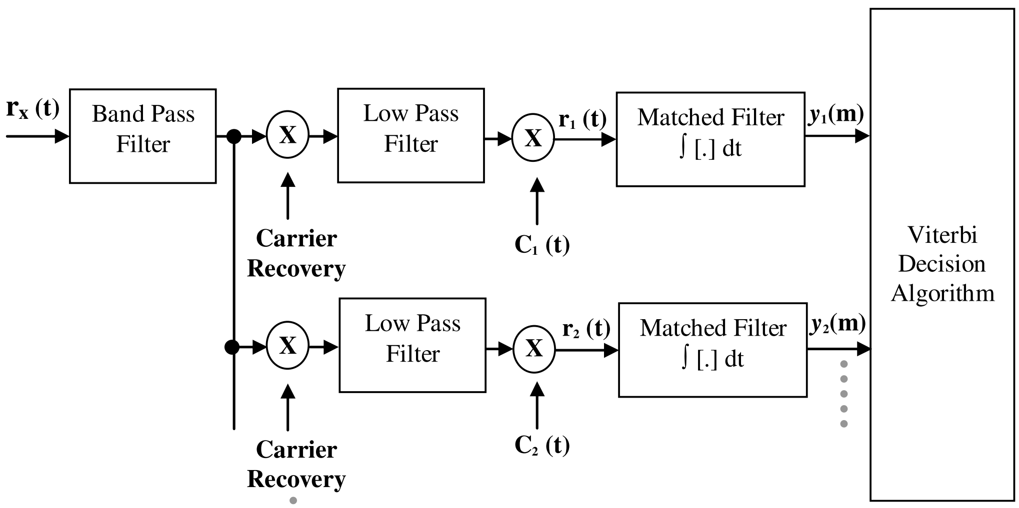

Fig.1. Maximum likelihood optimum receiver

$$y_2(m) = \frac{1}{T}\left\{\int_{2+(m)T}^{2+(m+1)T} r_2(t)C_2(t-\tau_2)dt\right\} \tag{2}$$

The received signal $r_1(t)$ and $r_2(t)$ can be expressed as follows:

$$r_1(t) = \left(E_{C_1}\right)^{0.5}\sum_{i=-M}^{M} b_1(i)C_1(t-iT_b) \tag{3}$$

$$r_2(t) = \left(E_{C_2}\right)^{0.5}\sum_{i=-M}^{M} b_2(i)C_2(t-iT_b-\tau_2) \tag{4}$$

Where E_{C_1} and E_{C_2} represent the original bit energy of the received signals. Substitute (3) and (4) as an individual equation into (1) and (2), respectively, and we get

$$y_1(m)=\frac{1}{T}\left\{\int_{(m)T}^{(m+1)T}\left(E_{C_1}\right)^{0.5}\left\{\sum_{i=-M}^{M}\left\{b_1(i)C_1\left(t-iT_b\right)\right\}\right\}C_1(t)dt\right\} \tag{5}$$

Substitute equation (5) as an individual equation into (1), we have

$$y_2(m)=\frac{1}{T}\left\{\int_{(m)T}^{(m+1)T}\left(E_{C_2}\right)^{0.5}\left\{\sum_{i=-M}^{M}\left\{b_2(i)C_2\left(t-iT_b-\tau_2\right)\right\}\right\}C_2\left(t-\tau_2\right)dt\right\} \tag{6}$$

Substitute equation (5) as an individual equation into (2), we have

$$y_2(m)=\frac{1}{T}\left\{\begin{array}{l}\int_{(m)T}^{(m+1)T}\left(E_{C_2}\right)^{0.5}\left\{\sum_{i=-M}^{M}\left\{b_2(i)C_2\left(t-iT_b-\tau_2\right)\right\}\right\}\times \\ C_2\left(t-\tau_2\right)dt\end{array}\right\} \tag{7}$$

After performing integration over the given interval, we get the following results with the noise components as well as the cross correlation of signature waveforms

$$\begin{array}{l} y_1(m) = \left(E_{C_1}\right)^{0.5} b_1(m) + \left(E_{C_2}\right)^{0.5} b_2(m-1)\rho_1 + \\ \left(E_{C_2}\right)^{0.5} b_2(m)\rho_0 + \left(E_{C_2}\right)^{0.5} b_2(m+1)\rho_{-1} + n_1(m)\end{array} \tag{8}$$

$$\begin{array}{l} y_2(m) = \left(E_{C_2}\right)^{0.5} b_2(m) + \left(E_{C_1}\right)^{0.5} b_1(m-1)\rho_1 + \\ \left(E_{C_1}\right)^{0.5} b_1(m)\rho_0 + \left(E_{C_1}\right)^{0.5} b_1(m+1)\rho_{-1} + n_2(m)\end{array} \tag{9}$$

Where coefficients $b_1(m)$ and $b_2(m)$ represent MAI, $\rho_{-1/0/+1}$ are cross-correlations of signature waveforms, and $n_1(m)$ and $n_2(m)$ represent the minimum noise components. Since the channel is LTI, the probability of unwanted noise is minimum.

These symbols can now be decoded using a maximum likelihood Viterbi decision algorithm. Viterbi algorithm can be used to detect these signals in much the same way as convolution codes. This algorithm makes decision over a finite window of sampling instants rather than waiting for all the data to be received [2, 7]. The above derivation can be extended from two users to *K* number of users. The number of operations performed in the Viterbi algorithm is proportional to the number of decision states, and the number of decision states is exponential with respect to the total number of users. In other words, the computational complexity grows exponentially with respect to the total number of users. The asymptotic computational complexity of this algorithm can be approximated as: $O(2)^{K}$

C. *Proposed Transformation Matrix Technique*

According to original Verdu's algorithm, the outputs of the matched filter y_1(m) and y_2(m)*, and* can be considered as a single output *y(m).* In order to minimize the noise components and to maximize the received demodulated bits, we can transform the output of the matched filter, and this transformation can be expressed as follows: $y(m)$ = Tb + n where T represents the transformation matrix, $b_k \in \{\pm 1\}$ and n represents the noise components. In addition, if vectors are regarded as points in K-dimensional space, then the vectors constitute the constellation diagram that has K total points.

This constellation diagram can be mathematically expressed as:

$$\aleph = \{\mathrm{T}b\} \; where \; b \in \{-1,+1\}$$

We use the above equation as a fundamental equation of the proposed system. According to the detection[1] rule, the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as $\int$) that can only intersect each other at the following points: $\aleph = \{\mathrm{Tb}\}_{b \in \{-1, 1\}}{}^{K} \backslash \int$

Fig. 2 shows the constellation diagram that consists of three different vectors (lines) with the original vector '**X**' that represents the collective complexity of the receiver. Q, R, and S represent vectors or transformation points within the coverage area of a cellular network as shown in Fig. 2. In addition, $Q^\neg$, $R^\neg$, and $S^\neg$ represent the computational complexity of each individual transformation point. In order to compute the collective computational complexity of the optimum receiver, it is essential to determine the complexity of each individual transformation point. The computational complexity of each individual transformation point represents by $X^\neg$ of the transformation point which is equal to the collective complexity of $Q^\neg$, $R^\neg$, and $S^\neg$. In order to derive the value of the original vector **X**, we need to perform the following derivations. We consider the original vector with respect to each transmitted symbol or bit (i.e., the number of lines or vectors in the constellation diagram).

$$X^\neg Q = Xi^\neg = \left(XQ_i + XR_j + XS_K\right) i^\neg$$

$$= XQii^\neg + XRji^\neg + XSki^\neg$$

$$X^\neg R = Xj^\neg = \left(XQ_i + XR_j + XS_K\right) j^\neg$$

$$= XQij^\neg + XRjj^\neg + XSkj^\neg$$

$$X^\neg S = Xk^\neg = \left(XQ_i + XR_j + XS_K\right) k^\neg$$

$$= XQik^\neg + XRjk^\neg + XSkk^\neg$$

The following system can be derived from the above equations:

$$\begin{pmatrix} X^\neg Q \\ X^\neg R \\ X^\neg S \end{pmatrix} = \begin{pmatrix} ii^\neg & ji^\neg & ki^\neg \\ ij^\neg & jj^\neg & kj^\neg \\ ik^\neg & jk^\neg & kk^\neg \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \tag{10}$$

Equation (10) represents the following: *QRS* with the unit vectors *i*, *j*, and *k;* $X^\neg Q$, $X^\neg R$, and $X^\neg S$ with the inverse of the unit vectors $i^\neg, j^\neg$, and $k^\neg$. The second matrix on the right hand side of equation (10) represents b, where as the first matrix on the right hand side of equation (10) represents the actual transformation matrix. Therefore, the transformation matrix from the global reference points (which could be *Q*, *R*, or *S*) to a particular local reference point can now be derived from equation (10)

$$T_{L/G} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} = \begin{pmatrix} X^\neg Q \\ X^\neg R \\ X^\neg S \end{pmatrix} \tag{11}$$

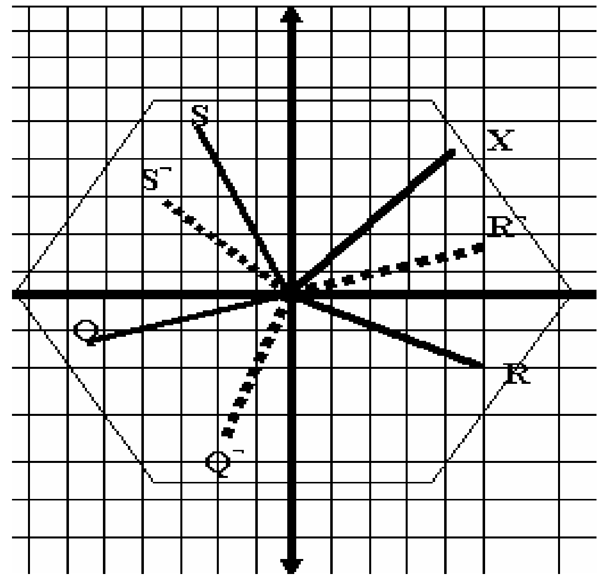

Fig.2. A constellation diagram consisting of three different parameters

Equation (11) can also be written as:

$$T_{L/G} = \begin{pmatrix} ii^\neg & ji^\neg & ki^\neg \\ ij^\neg & jj^\neg & kj^\neg \\ ik^\neg & jk^\neg & kk^\neg \end{pmatrix} \tag{12}$$

In equation (12), the dot products of the unit vectors of the two reference points are in fact the same as the unit vectors of the inverse transformation matrix of equation (11). We need to compute the locations of the actual transformation points described in equations (11) and (12). Let the unit vectors for the local reference point be:

$$\begin{aligned} i^\neg &= \left[t_{11} i, t_{12} j, t_{13} k \right] \\ j^\neg &= \left[t_{21} i, t_{22} j, t_{23} k \right] \\ k^\neg &= \left[t_{31} i, t_{32} j, t_{33} k \right] \end{aligned} \tag{13}$$

Since, $i^\neg \left(i + j + k\right) = i^\neg$, where $\left(i + j + k\right) = 1$. The same is true for the rest of the unit vectors. Therefore, equation (13) can be rewritten as:

$$\begin{aligned} i^\neg &= \left[t_{11}, t_{12}, t_{13} \right] \\ J^\neg &= \left[t_{21}, t_{22}, t_{23} \right] \\ k^\neg &= \left[t_{31}, t_{32}, t_{33} \right] \end{aligned} \tag{14}$$

By substituting the values of $i^\neg, j^\neg$, and $k^\neg$ from equation (14) into equation (12), we obtain

$$T_{L/G} = \begin{pmatrix} i(t_{11}i + t_{12}j + t_{13}k) & j(t_{11}i + t_{12}j + t_{13}k) & k(t_{11}i + t_{12}j + t_{13}k) \\ i(t_{21}i + t_{22}j + t_{23}k) & j(t_{21}i + t_{22}j + t_{23}k) & k(t_{21}i + t_{22}j + t_{23}k) \\ i(t_{31}i + t_{32}j + t_{33}k) & j(t_{31}i + t_{32}j + t_{33}k) & k(t_{31}i + t_{32}j + t_{33}k) \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \tag{15}$$

Substituting $\boldsymbol{T_{L/G}}$ from equation (15) into equation (11), yields

$$\begin{pmatrix} X \neg Q \\ X \neg R \\ X \neg S \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \begin{pmatrix} X Q \\ X R \\ X S \end{pmatrix} \qquad (16)$$

Equation (16) corresponds to the following standard equation used for computing the computational complexity at the receiving end:

$$\aleph = \{\mathrm{Tb}\} \quad b \in \{-1, +1\}^k$$

If the target of one transformation $(U : Q \to R)$ is the same as the source of other transformation $(T : R \to S)$, then we can combine two or more transformations and form the following composition: $TU : Q \to S,\ TU(Q) = T(U(Q))$. This composition can be used to derive the collective computational complexity at the receiving end using equation (16). Since we assumed that the transmitted signals are modulated using BPSK which can at most use 1 bit out of 2 bits (that is, $b_k \in \{\pm 1\}$), consider the following set of transformation points to approximate the number of demodulated received bits that need to search out by decision algorithm:

$$\aleph = \left(\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \right)^K$$

$$+ \left(\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} \right)^K$$

Using (16), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated transformation points. The set of transformation points correspond the actual location with in the transformation matrix as shown in equation (16). The entire procedure for computing the number of demodulated bits that need to search out by the decision algorithm can be used to approximate the number of most correlated signals for any given set of transformation points. This is because, we need to check weather or not the transformation points are closest to either (+1, +1) or (-1, -1). The decision regions or the coordinates where the transformation points lie for (+1, +1) and (-1, -1) are simply the corresponding transformation matrixes that store the patterns of their occurrences. In other words, the second matrix on the right hand side of (16) requires a comprehensive search of at most 5^K demodulated bits that indirectly correspond to one or more users. The minimum search performed by the decision algorithm is conducted if the transformation points exist within the incorrect region. Since the minimum search saves computation by one degree, the decision algorithm has to search at least 4^k demodulated bits. The average number of computations required by a system on any given set always exists between the maximum and the minimum number of computations performed in each operational cycle. This implies that the total number of demodulated bits that need to search out by the decision algorithm can not exceed by $5^k - 4^k$. In other words, the total number of most correlated pairs is

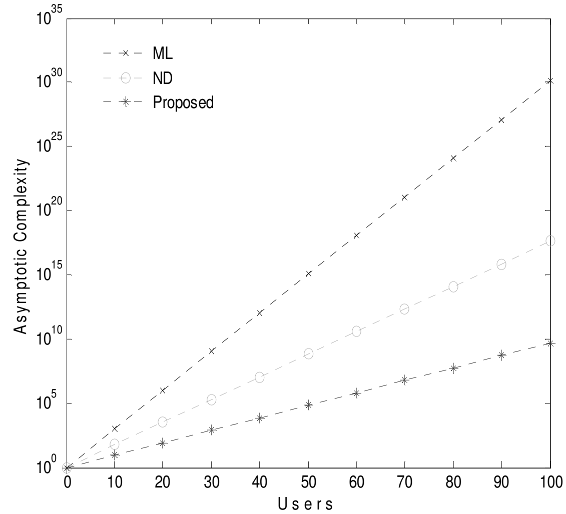

Fig.3. The asymptotic computational complexities versus intermediate number of users

upper bounded by $5^k - 4^k$. Using Newton approximation method given in MATLAB, we can directly come to an approximation as $O(5/4)^K$

IV. Performance Analysis

Fig. 3 shows the computational complexities for a network that consists of 100 users. As we can see that the proposed system for a network of 100 users requires fewer computations as compare to the ML and the ND system. In addition, the proposed algorithm greatly reduces the unnecessary computations involve in signal detection by storing the pattern of occurrence of the demodulated bits in the transformation matrix and uses it only on those coordinates or decision regions which are most likely lead to an incorrect decision.

Simulation results show that the proposed technique performs better than the ML and the ND algorithms for all values of BER. Figure 4 and 5 show a plot of three BER

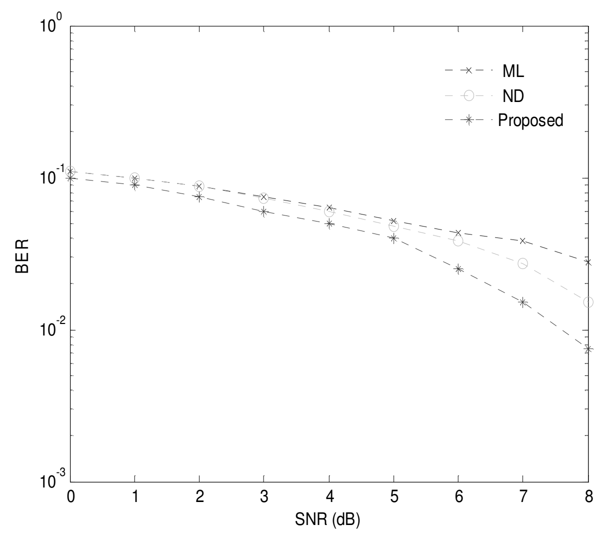

Fig.4. BER versus SNR (0<dB<9) curves

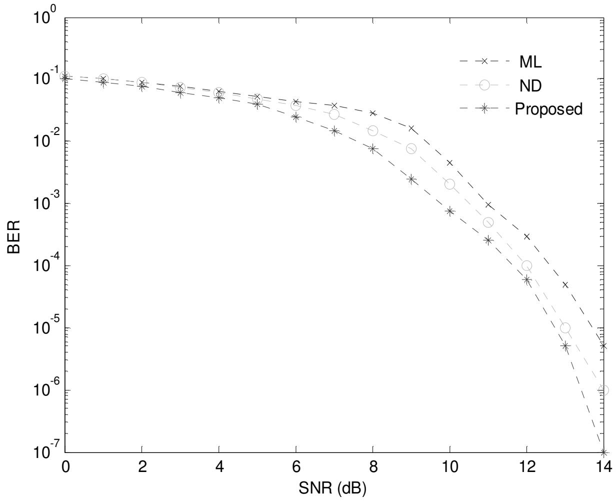

Fig.5. BER versus SNR (0<dB<14) curves

versus SNR curves. These curves were plotted in an AWGN channel for a small range of users. It should be noted that the BER performance of the proposed technique is always better than the ML and the ND algorithms as shown in Fig. 4. For the first few values of SNR, the ND algorithm almost approaches the ML algorithm whereas the proposed technique still maintains a reasonable performance difference. It can be seen in Fig. 4 that the proposed technique achieves less than 10^{-2} BER for SNR = 8 dB which is quite closed to the required reasonable BER performance for a voice communication system. For small values of SNR, the BER for these three algorithms is almost equal, but as we increase the value of SNR, typically more than 10 dB, one can clearly observe the difference in the BER performance.

The former result demonstrates a slight improvement over the BER performance shown in Fig. 4 for all SNR values above 9 dB. Even for small values of SNR, the proposed technique gives better performance than the ML and the ND algorithms. As the value of SNR increases, the BER performance of the proposed technique over the ND and the ML algorithms becomes more and more substantial because the probability of having more divergent values of SNR increases. It can also be noticed in Fig. 5 that the proposed algorithm achieves less that 10^{-3} BER for SNR = 10 dB which is more than what we desire for a voice communication system. Furthermore, the proposed technique achieves 10^{-7} BER performance for SNR = 14 dB as shown in Fig. 5. This is more than an acceptable value of BER for data communication such as FTP and is quite close to the desired value of BER (typically 10^{-8} BER is required) for high fidelity digital audio systems.

V. CONCLUSION

We proposed a new transformation matrix technique that can be used to significantly reduce the asymptotic computational complexity of multiuser receiver. We also presented a mathematical model which verifies the implementation of the transformation matrix technique. Furthermore, our simulation results show that the proposed technique gives better BER performance than the other multiuser algorithms. This reduction in BER would likely minimize the packet loss and maximize the overall network data throughput for a DS-CDMA system. For the future work, we will present the mathematical model to quantify the data throughput with the minimum packet loss for the proposed transformation matrix technique.

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