Automatic frequency extraction using sinusoidal approximation and wavelet transform

J. Rafiee, M.A. Rafiee, N. Prause, P.W. Tse

***Abstract*— This paper introduces a novel time-frequency-based pattern recognition technique for gear fault identification using time-domain analysis of continuous wavelet transform. It, moreover, presents a new approximation method for gearbox vibration signals. Sinusoidal approximation approximates sinusoidal components of noisy vibration signals generated from gearboxes using autocorrelation of wavelet coefficients. First, the shortcomings of continuous wavelet transform have been ameliorated using autocorrelation function. Afterwards, the autocorrelation of wavelet coefficients have been introduced as an original feature for monitoring purposes of gearbox diagnostics. Finally, after choosing a precise preprocessor, a sinusoidal function has been further considered to approximate the periodic waveforms of autocorrelation coefficients for both normal and faulty gearboxes. As reported in this paper, gearbox vibration signals with non-stationary characteristics can be properly classified with minimal loss of information in frequency contents using the sinusoidal approximation of generated autocorrelation waveforms of wavelet coefficients.**

***Index Terms*—wavelet, signal processing, autocorrelation, sinusoidal approximation, Daubechies 44, gear.**

I. Introduction

Gearboxes are common mechanical systems in industry. Frequent failures of their gears make them challenging systems to monitor adequately. To reliably monitor vibration signals [1-3], various techniques have been studied including time- [4], frequency- [5], and time-frequency-domain [6]. Wavelet transform (WT) [7], which has been advancing in the past two decades, outweighs other time-frequency methods. However, major concern to diagnose machine faults using wavelets is finding a feature consistently identifies the signal features with small-sized configurations. Continuous wavelet transform generates Continuous Wavelet Coefficients (CWC) in different scales. These scales include all necessary information for monitoring; thus, when resampling them for fault identification, significant information may not be lost.

On the other hand, vibration signals of rotating machines contain natural and defect frequencies due to periodic behaviors of the machine in which the extraction of significant frequencies by a small-size feature is still a challenge. Identifying a common feature applicable across machines is complex. As most of the vibration signals generated from practical industrial cases are non-stationary, a proper feature that could be used for different dataset is of great consequence. Several papers [8-11] have used wavelet transform [12]. However, the improvement of time-domain analysis of wavelet transform to cover wavelet draw backs is scarce [13].

In this research, a new technique was tested to compress data while maintaining information based on time-series analysis of Continuous Wavelet Transform (CWT). Data reduction with essential feature retention was accomplished using the autocorrelation of CWC of gearbox vibration signals. This approach offers advantages over the previous approach [13, 14]. Furthermore, a sinusoidal summation function to approximate the periodic trends of autocorrelation of CWC with satisfactory preciseness was designed.

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II. THEORIES AND MODUS OPERANDI

The procedure included:

1. Collecting vibration signals for three types of defect gears plus normal gears from a motorcycle gearbox.
2. Piecewise cubic spline interpolation [15] to synchronize the vibration signals.
3. Calculating continuous wavelet coefficients of synchronized vibration signals (CWC-SVS) to 4^{th} decomposition level.
4. Autocorrelation of CWC-SVS was determined.
5. Comparing the power spectrum density (PSD) of CWC-SVS with and without autocorrelation.
6. Frequency bands from the PSD of autocorrelated CWC-SVS were used to identify vital features to distinguish gearboxes operating under normal and abnormal conditions.

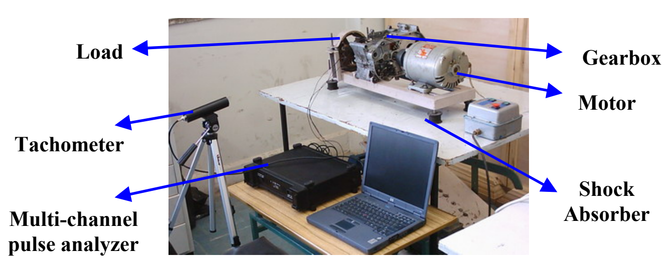


Fig. 1. (A) Experimental set-up

7. The autocorrelation of CWC-SVS was approximated by sinusoidal summation function to test the accuracy of reconstructing the original signal for normal and faulty gear patterns.

A. Data acquisition

A four-speed gearbox was run to provide vibration data. It consisted of a driven motor with a constant nominal rotation speed of 1420 rpm, its load mechanism, four shock absorbers under the bases of test-bed, and the gearbox. The box for the faulted and driven gear possessed 24 and 29 teeth, respectively. As shown in Figure 1, a multi-channel 'Pulse' analyzer system, triaxial accelerometers and a tachometer were used. The vibration signals were collected by mounting the accelerometer on the outer surface of the gearbox's case near the input shaft of the gearbox. Three different fault conditions were tested (slight-worn, medium-worn, and broken-tooth of a spur gear). For evaluating the accuracy of these techniques, two similar models of gear faults (slightly worn, medium worn) were compared. The real rotational speed of the motor was measured by the tachometer to compensate for fluctuations of the system due to uncertainties of the load. The rotating speed of the motor was 24.05 Hz. Based on this and the gear ratio, the driving gear mesh frequency and driven gear mesh frequency were calculated as 577.4 Hz and 697.7 Hz respectively. The sampling rate was set at 16,384 Hz.

B. Preprocessing of gearbox vibration signals

Since gearbox signals are non-synchronous, the number of data-points per each shaft revolution may not be equal due to shaft speed fluctuations and/or load. Piecewise cubic interpolation was applied to resample the data to a normal time base prior to signal analysis.

The basic theory of Continuous Wavelet Transform (CWT) for applications in condition monitoring is well-characterized in published literature [12]. Therefore, after synchronizing the raw gear vibration signals, the CWT and autocorrelation were applied to the synchronized signals and produced CWC-SVS. The db44 [16] was used as the mother wavelet function.

C. Results and Discussion

For a signal $x(t)$, the autocorrelation function [17] is the average value of the product $x(t).x(t+\tau)$, where τ is time delay. Formally, the autocorrelation function, $R_x(\tau)$, is defined as;

$$R_x(\tau) \equiv E[x(t).x(t+\tau)] = \lim_{T\to\infty}\int_0^T x(t)\; x(t+\tau)\,dt \qquad (1)$$

Mean and variance of this function are independent of time. Therefore,

$$E[x(t)] = E[x(t+\tau)] = x' \qquad (2)$$

and $\sigma^2_{x(t)} = \sigma^2_{x(t+\tau)} = \sigma^2_x = E[x^2(t)] - x'^2$ (3)

The autocorrelation coefficient is defined as:

$$\rho_x x(\tau) \equiv \frac{E\{[x(t)-x'].[x(t+\tau)-x']\}}{\sigma_x^2} \qquad (4)$$

which is expanded as Equation 5 as follows:

$$\rho_x x(\tau) \equiv \frac{E\{[x(t).x(t+\tau)] - x'E[x(t+\tau)] - x'E[x(t)] + x'^2\}}{\sigma_x^2}$$

Substitution of Equations (1) and (2) into Equation (5) leads to an expression that relates the autocorrelation function to its coefficients:

$$\rho_{xx}(\tau) \equiv \frac{R_x(\tau) - x'^2}{\sigma_x^2} \qquad (6)$$

or $R_x(\tau) = \rho_{xx}(\tau)\,\sigma_x^2 + x'^2$ (7)

Some limits can be placed on the value of $R_x(\tau)$. Because $-1 \le \rho_{xx}(\tau) \le 1$, $R_x(\tau)$ is bounded as:

$$-\sigma_x^2 + x'^2 \le R_x(\tau) \le \sigma_x^2 + x'^2 \qquad (8)$$

The variance of x is expressed in terms of the expectation of x^2, $E[x^2]$, and the square of the mean of x, x'^2:

$$\sigma^2 = E[x^2] - x'^2 \qquad (9)$$

Based on Equation (1), in Equation (8) with regard to Equation (9) we have:

$$R_x(0) = E[x^2] = \sigma^2 + x'^2 \qquad (10)$$

So, the maximum value that $R_x(\tau)$ can have is $E[x^2]$, which occurs at $\tau = 0$. Using Equation (9) and the definition of the autocorrelation coefficients as in Equation (6),

$$\rho_{xx}(0) = 1 \qquad (11)$$

Further, when $\tau \to \infty$ there is less correlation between $x(t)$ and $x(t+\tau)$, because $x(t)$ is the signal of a random variable. That is,

$$\rho_{xx}(\tau \to \infty) = 0 \qquad (12)$$

which indicates that:

$$R_x(\tau \to \infty) = x'^2 \qquad (13)$$

a value equal to the limiting value, which signifies, there is no correlation.

Autocorrelated CWC-SVS may, then, be able to reduce the size of a non-stationary signal while minimizing information loss. This would offer a significant advantage over past approaches.

D. Sinusoidal Approximation

A sinusoidal summation function to assess signal reconstruction is defined as:

$$f(x) \equiv \sum_{i=1}^{n} a_i \sin(b_i\, x + c_i) \qquad (14)$$

where a_i, b_i, and c_i are constant coefficients. In this paper, n is 8. The purpose of gearbox monitoring is to identify a small-structure pattern to differentiate the conditions. It is obvious that the more n, the better approximation we have. However, $f(x)$ consisting of 8 terms can approximate all gearbox conditions with high accuracy for each scale. Simply, a 3 constant coefficient in this function can compress the sub-signals to a meaningful and reliable approximation with 3×8 elements for each 16 scale. To verify approximation accuracy, the following common statistical criteria have been considered in this research:

SSE: The sum of the squares of the error measures the total deviation of the response values from the fit to the response values:

$$SSE \equiv \sum_{i=1}^{N}(x_i - x_{ci})^2 \quad (15)$$

A value closer to 0 indicates that the approximation has a smaller random error component.

R.S: R-square is defined as the ratio of the sum of squares of the regression (SSR) and the total sum of squares (SST):

$$SSR \equiv \sum_{i=1}^{N} w_i(\hat{y}_i - \bar{y})^2 \quad (16)$$

SST is also called the sum of squares about the mean, and is defined as:

$$SST \equiv \sum_{i=1}^{N} w_i(y_i - \bar{y})^2 \quad (17)$$

where SST = SSR + SSE. Given these definitions, R-square is expressed as:

$$R.S = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \quad (18)$$

R-square can take on any value between 0 and 1, with a value closer to 1 indicating that a greater proportion of variance is accounted for by the approximation.

A.R.S: Degrees of freedom adjusted R-Square uses the R-square and adjusts it based on the residual degrees of freedom. The residual degree of freedom is determined as:

$$v = n - m \quad (19)$$

where n is the number of response values and m is the number of fitted coefficients. The residual degree of freedom indicates the number of independent pieces of information including the n data-points that are required to determine the sum of squares. The degrees of freedom are increased by the number of such parameters.

A.R.S can take on any value less than or equal to 1, with a value closer to 1 indicating a better fit. Negative values can occur when the model contains terms that do not help to predict the response.

RMSE: Root Mean Squared Error is also known as the fit standard error which can estimate the standard deviation of the random components in the data:

$$RMSE = \sqrt{MSE} \quad (21)$$

where MSE is the mean square error.

A lower RMSEA indicates that the reduced signal better matches the original, thus minimizing loss of information.

E. Autocorrelated CWC-SVS for fault classification

In autocorrelation, the signal is correlated with itself at varying time lags. Essential information such as the repeatability of the signal can be gathered. Autocorrelation makes the time-periodic trends of the signal explicit by pinpointing regular patterns in defective gears, especially worn gears. Due to many components of the signals, CWC-SVS is used to divide the signal into these regular patterns. Next, the autocorrelations of each scale of CWC-SVS with itself were determined for all four conditions (normal, slight-worn, medium-worn and broken-tooth). The autocorrelation was calculated for each of four levels of decomposition. In Figures 2 and 3, eight of 2^4 representing the scales from (4,8) to (4,15) of the autocorrelated CWC-SVS under normal condition and medium-worn have been displayed.

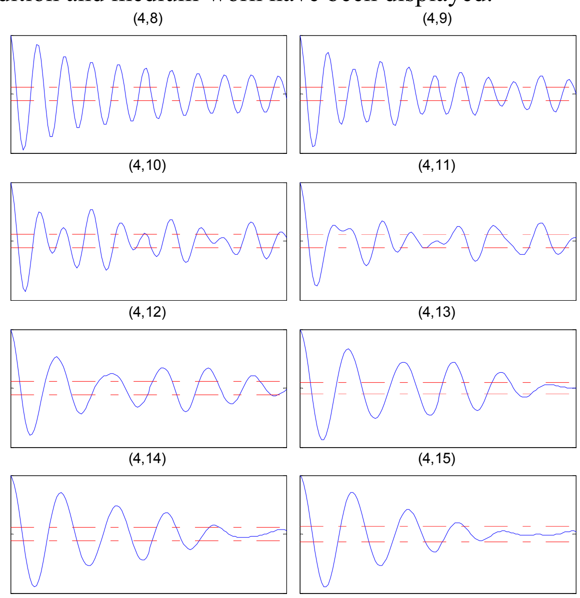


Fig. 2. Autocorrelation plot of CWC-SVS for normal gearbox (decomposition level, scale) in title

The plots depict the variation in the decomposed components of the signals under different gear health conditions. These highlight the differences between each condition across sub-signals, which permits simple classification of different gear conditions.

In vibration-based gear fault diagnosis, the frequency-domain based PSD, which contains meshing frequencies, their harmonics, and sidebands, is commonly used to identify the gear related frequency contents. PSD of raw signals collected from the gearbox operating under normal condition and broken-tooth is depicted in Figures 4 and 5, respectively, with f as tooth meshing frequency and $2f$, $3f$, etc as its harmonics. To verify that there is no significant loss of information, after applying autocorrelation to the CWC-SVS, a few of the PSDs generated at every quarterly scale of the 4^{th} level are shown in Figure 6 for comparison to those generated by CWC-SVS only.

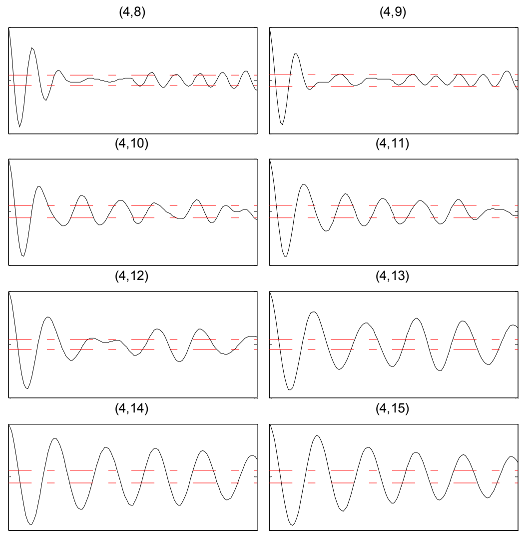


Fig. 3. Autocorrelation plot of CWC-SVS for medium-worn gear (decomposition level, scale) in title

In machine condition monitoring, major concern remains: Is the pattern used for recognition applicable for different sample signals extracted in various times from machine given that the signals are embedded with non-stationary attribute? The autocorrelation of CWC-SVS responds to this concern which is generalizable across signals with minimal fluctuations. For example, Figure 7 shows the results from 50 revolutions of rotation when the gearbox was in a slightly-worn condition. Note that the variation is very small, which indicates that the proposed method is robust for non-stationary signals.

Autocorrelation function has been proven reliability to parse randomness of data. For the gearbox vibration data, the number of time lags for gearbox vibration signals can be limited to less than 30, because this sufficiently identifies the randomness contained in the data. In the present study, 125 time lags were used as qualitatively determined to provide the better account for the signals. The reason for such a time lag was to permit not only classification of random portions of the data, but also to reduce the size of CWC-SVS to almost 1/6 of its original size. In general, the larger number of lags, the higher accuracy. The large number of lags will lead to large-size feature.

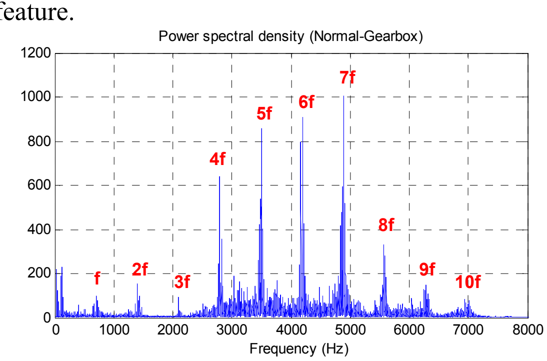


Fig. 4. PSD of raw vibration signal with f as tooth meshing frequency and 2f, 3f etc. as its harmonics (Normal Gearbox)

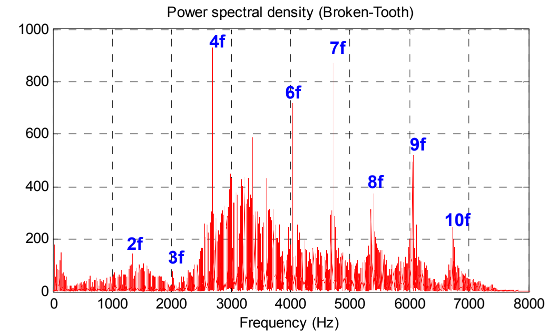


Fig. 5. PSD of raw vibration signal (broken-tooth)

Figure 8 shows 8 PSD plots generated by high scale of 4th decomposition level of autocorrelated CWC-SVS's results. Note that the X-axis is the number of lags (125). The autocorrelation of CWC-SVS will present 125-elements vector for each scale.

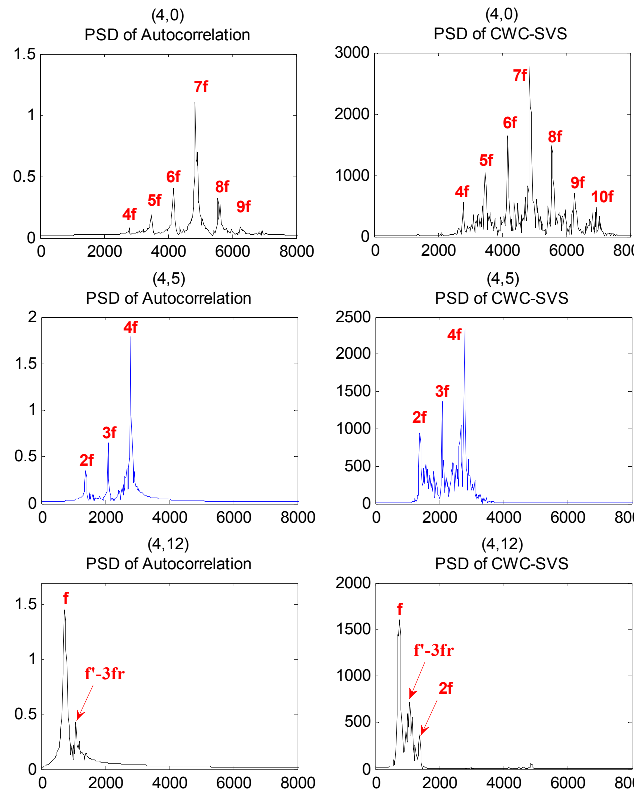


Fig. 6. The comparison of the PSDs generated by autocorrelation of CWC-SVS and those generated by CWC-SVS of the gearbox normal condition

To reflect the number of lags, each PSD plot is presented for 125 frequency points rather than up to 8 kHz. It means that instead of distributing the frequency from 0 to 8000 Hz, as in Figure 4 and 5, the horizontal axis has been distributed evenly from 1 to 125 (the number of lags) units. In other words, the horizontal axis is indirectly proportional to time. The vertical axes reflect the different gear conditions, including 10 sample signals for each of the four conditions. The third axis displays the magnitude of autocorrelations of CWC-SVS (shown as different color scales).

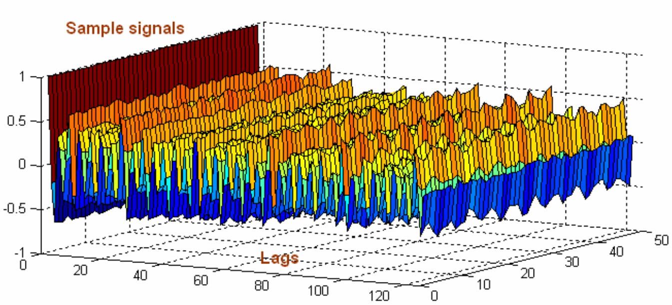
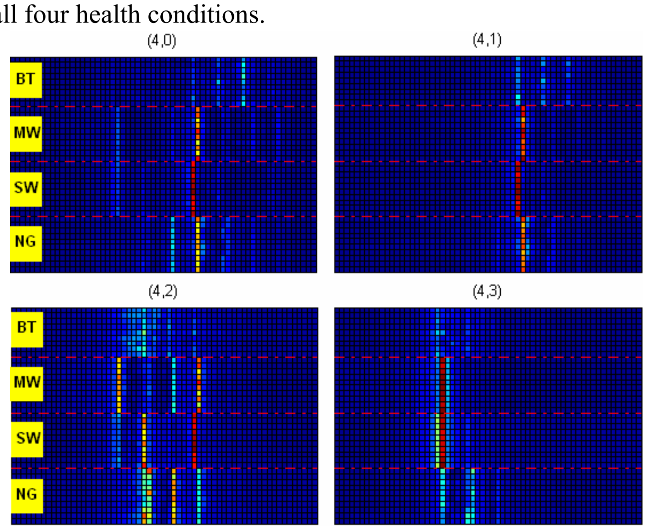


Fig. 7. Autocorrelation of CWC-SVS in 50 revolutions of the shaft [Slight-worn gear / scale (4, 0)]

At high scale (low-frequency) from (4, 8) to (4, 15), the dominant frequency as well as its sidebands of each health condition can be revealed clearly. However, classifying the four health conditions using these scales are difficult as their features are located adjacently. Whilst in low scale (high frequency), such as in the scale of (4, 2), the features for each of four health conditions can be easily distinguishable (see Figure 8). The clear discriminability of these characteristic frequencies and patterns will permit useful future automated algorithms for feature extraction.

F. Sinusoidal Approximation of Gear Vibration Signals

Using the aforementioned statistical evaluation methods, the preciseness of the sinusoidal approximation was tested across four conditions of the gearbox. The original values of the autocorrelation of CWC-SVS (displayed as continuous curves) as compared to approximated values (displayed as discrete spots) for some arbitrary selected scales under the medium-worn health condition are shown in Figure 9. Note that the approximation can follow the waveforms of autocorrelated CWC-SVS closely, particularly at higher scales. As an example, the results of statistical evaluation are tabulated for medium worn in Tables 1. From the results shown in Figure 9 and Table 1, sinusoidal summation function with 8 terms for each scale was able to accurately approximate the waveforms generated by autocorrelation of CWC-SVS for all four health conditions.



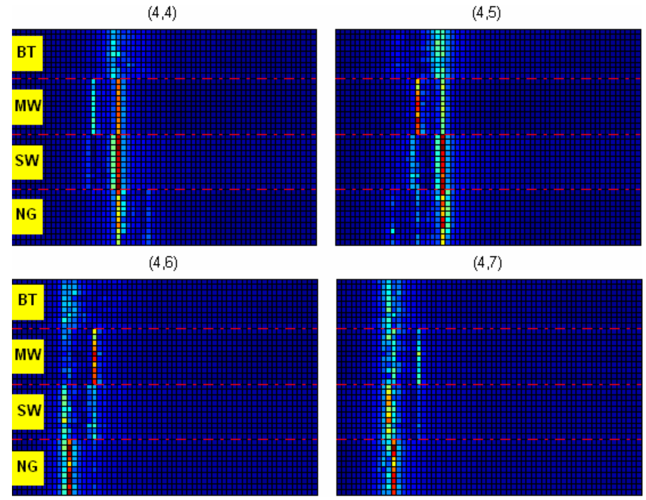
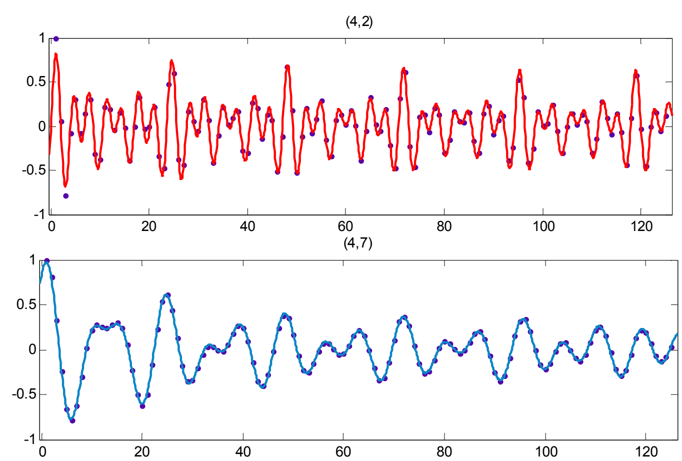


Fig. 8. Automatic frequency extraction of vibration signals by means of PSD of autocorrelation of CWC-SVS distributed on the number of lags (125) for 10 sample signals of each condition (high scales)

Lower scales provide much better fit to the data in statistical comparisons (see Table 1). This likely is attributable to the autocorrelation of CWC-SVS in higher scales possessesing more different frequency contents in comparison with lower scales. In these figures, the frequency components of autocorrelation of CWC-SVS in high scales are more than those in low scales. Statistical comparisons support this observation, clearly showing the superiority of sinusoidal approximation in comparison to low scales (see Table 1). Although the goodness of the fitness is satisfactory by using only a sinusoidal summation function with 8 terms, the number of sinusoidal terms (n) in Equation 14 can be increased for more complex waveforms. The authors believe that the sinusoidal approximation might be the base of several methods used in pattern recognition. The proposed approximation function also might be applicable for other defects, such as bearing defects, because the bearing faulty signals are impulsive in nature, similar to gear faulty signals.



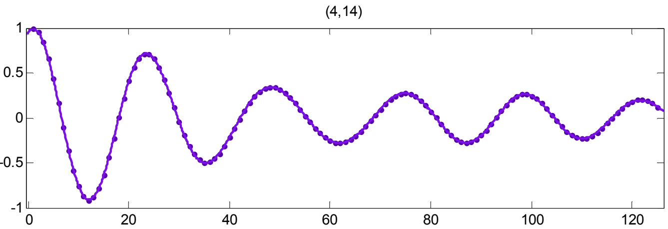


Fig.9. Approximation of autocorrelation of CWC-SVS for Medium-Worn

Table 1. Goodness of approximation for Medium-Worn Gearbox

Criteria	4, 0	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6	4, 7
SSE	0.34	0.14	0.12	0.06	0.09	0.24	0.24	0.00
R.S.	0.97	0.99	0.99	1.00	0.99	0.98	0.99	1.00
A.R.S.	0.97	0.99	0.99	1.00	0.99	0.98	0.98	1.00
RMSE	0.06	0.04	0.03	0.03	0.03	0.05	0.05	0.00
Criteria	4, 8	4, 9	4, 10	4, 11	4, 12	4, 13	4, 14	4, 15
SSE	0.02	0.03	0.11	0.00	0.00	0.00	0.00	0.00
R.S.	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
A.R.S.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
RMSE	0.01	0.02	0.03	0.00	0.00	0.00	0.00	0.01

III. CONCLUSION

1. Autocorrelation of CWC-SVS is a suitable method for efficiently reducing and analyzing non-stationary signals in machine condition monitoring.
2. The sinusoidal summation function can approximate the waveforms generated by autocorrelation of CWC-SVS for normal gearbox as well as other defective gears satisfactorily. The function achieved proper approximation even when the waveforms were different from one condition to another condition, as they possess different frequency contents of vibration signals. The proposed algorithm might be the base of feature extraction in machine condition monitoring such that the meaningful approximation coefficients with the small-size attribute can be realized.
3. Last but not least, Daubechies 44 is one disscussable mother wavelet used in this research.

IV. ACKNOWLEDGEMENT

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