Application of Daubechies 44 in Machine Fault Diagnostics

J. Rafiee, M.A. Rafiee, N. Prause, P.W. Tse

***Abstract*— This research focuses on the application of Daubechies 44 (db44) for gearbox vibration signals. Vibration signals of a sophisticated motorcycle gearbox system have been collected in four conditions: Normal Gearbox, Slight-Worn gear, Medium-Worn gear and Broken-Tooth gear. To monitor the gearbox failures, new features were introduced based on four statistical criteria that are standard deviation, variance, kurtosis, and fourth central moment of continuous wavelet coefficients of synchronized vibration signals (CWC-SVS). Variance of CWC-SVS was selected as the pattern for finding the most similar mother wavelet function across gear defects. Among 324 mother wavelet candidates, results show that Daubechies 44 (db44) has a distinctive pattern across gearbox vibrations even in comparison with the adjacent Daubechies functions (db43 and db45). In sum, drawbacks of the type of mother wavelet function in gearbox diagnostics have been developed in this research.**

***Index Terms*— Condition Monitoring, Mother Wavelet, Signal Processing, Daubechies 44 (db44), Gearbox, Vibration.**

I. Introduction

Machinery diagnostics is one type of condition monitoring often used to identify incipient faults. Gears are the machine component most challenging to identify faults. Between vibration [1] and acoustic signals, the former has outputs more amendable to analysis, and it is applicable in the noisy environment of industrial factories. A wide variety of techniques have been introduced for analyzing gear vibration signals. These can be separated into two groups: conventional and advance signal processing, and intelligent systems [2]. For example, FFT [3], Wigner–Ville distribution [4], wavelet transform [5], Hilbert-Huang transform [6], blind source separation [7], statistical signal analysis [8], and their combinations [9] could be in one cluster and ANN-based [10], GA-based [11], various similar classifiers [12], and other combined algorithms [11, 12] could possibly categorized in the second branch.

Wavelets transform (WT) is capable of processing stationary and non-stationary signals in both time and frequency domains. This important approach is increasingly used for condition monitoring. A remaining challenge using wavelets is the selection of a mother wavelet function, which is a significant part of wavelet analysis [13]. In this research, db44 was investigated in condition monitoring as compared with its unique performance against other mother wavelet functions.

A few researchers have sought to optimize selection of a mother wavelet. Tse et al [14] introduced a novel GA-based exact wavelet analysis for machine fault diagnosis to optimize the scale and translation factors for a single and predefined mother wavelet. They also extended their own technique for direct optimization of the wavelet coefficients. Ahuja et al. [15] introduced B-spline wavelet function for image sequence super-resolution. Subsequent to Flanders [16], Singh and Tiwari [17] and Brechet et al. [18] also proposed their own algorithms for optimal selection of mother wavelet in ElectroCardioGraphy (ECG). Landolsi [19] also studied a few mother wavelets for simulating optical pulse propagation. Moreno-Baron et al. [20] presented an intelligent system for quantification purposes in a voltammetric electronic tongue with regard to the mother wavelet functions as well as decomposition level of signal. Although Daubechies family wavelets have been applied in many papers [21, 22] in condition monitoring, these tend to use lower order db (db1 to db20) [23, 24]. The application of high order db is surprisingly rare [25]. The current research will show db 44, which is a higher-order db, distinctively characterizes patterns of gear vibration signals.

II. Proposed Technique

The progress of the research is as follows:

1- Vibration signals of a spur geared system were collected from a motorcycle gearbox system with three different gear defects.

2- To synchronize the vibration signals due to the load fluctuations, piecewise cubic spline interpolation was used to synchronize vibration signals (see Figure 1 of one shaft revolution). 50 samples for four gearbox conditions were scrutinized.

3- Continuous Wavelet Coefficients of Synchronized Vibration Signals (CWC-SVS) were determined in 4^{th} level of decomposition (16 numbers for each sample).

4- In 4th decomposition level, standard deviation, variance, kurtosis, and 4th central moment of CWC-SVS were determined by 324 different mother wavelets. The one most similar to all signals, db44 had the highest values for wavelet coefficients or the best similarity to gearbox vibration signals.

Manuscript received Oct 14, 2008. **J. Rafiee** is with Multidisciplinary Design Lab, Department of Mechanical, Aerospace & Nuclear Engineering at Rensselaer Polytechnic Institute, NY, USA. **M.A. Rafiee *** is with Mechanical System Lab, Department of Mechanical, Aerospace & Nuclear Engineering at RPI. (*corresponding author email: rafiem@rpi.edu). **N. Prause** is with the Dept. of Psychology at Idaho State University, USA. **P.W. Tse** is the Director of Smart Engineering Asset Management Lab at City University of Hong Kong.

Decomposition level is another issue in wavelet analysis. The simple formula is: the more level of decomposition we have, the better resolution on the signal. However, computational time and small-size feature are also two significant points. In this type of vibrations, all failures are obvious in 3rd level of decomposition. Nonetheless, for finding the best mother wavelet, level 4 was determined for more confidence since finding the best mother wavelet is an offline process for our system.

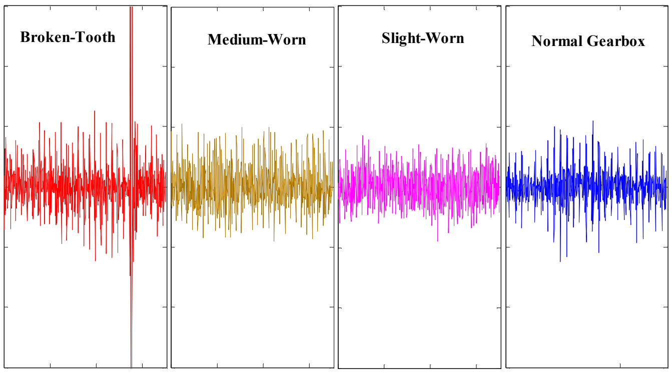

Fig. 1. One sample signal of different gearbox conditions

A. Experimental Data Acquisition

A four-speed motorcycle gearbox containing oil was used for data acquisition, driven at a constant rotation speed of 1420 RPM, load mechanism, multi-channel pulse analyzer data acquisition system, triaxial accelerometer, tachometer and four shock absorbers under the bases of test-bed. Vibration signals were collected by a mounted accelerometer on the surface of the gearbox's case near input shaft. Four conditions (Slightly-Worn, Medium-Worn, Broken Tooth, and Normal) were tested. A tachometer measured rotational speed and helped synchronize measures due to the variability of the load mechanism. Signals were sampled at 16384 Hz. Figure 2 depicts experimental set-up and related components.

B. Preprocessing of Gearbox Vibration Signals

Due to non-synchronous characteristic of gearbox signals, the number of data-points per each shaft revolution may not be equal because of the shaft speed fluctuations and the load. Piecewise Cubic Spline Interpolation (P.C.S.I.) [26] was used to resample the data to a normal time base prior to signal analysis.

Fig. 2. Experimental set-up

The length of sample signals that were not equal in gear dataset was synchronized by using P.C.S.I. with no loss of information.

C. Feature extraction using continuous wavelet transform

In machine condition monitoring, feature extraction is of paramount importance. Since there is no clear-cut equation for practical vibration signals, a reliable pattern is the fundamental factor. In this paper, four statistical features of CWC-SVS were investigated, and the variance of CWC-SVS was considered as the pattern for finding the best mother wavelet.

Continuous Wavelet Transform (CWT) has been used for machinery fault detection and diagnosis. A function $\psi \in L^2(R)\setminus\{0\}$ satisfying the admissibility condition:

$$c_\psi = 2\pi \int_{-\infty}^{+\infty} \left|\hat{\psi}(\xi)\right|^2 \frac{d\xi}{|\xi|} \prec \infty \quad (1)$$

is called a mother wavelet. Suppose ψ is the mother wavelet and $(a,b) \in R^* \times R$, the daughter wavelet of ψ will be defined as $W_\psi f : R^* \times R \rightarrow C$ where,

$$W_\psi(a,b) = \int_{-\infty}^{\infty} f(x)\overline{\psi}_{a,b}(x)\,d(x) \quad (2)$$

The CWT of a function $f \in L^2(R)$ with respect to the mother wavelet is defined by:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right) \quad (3)$$

Continuous Wavelet Coefficients (CWCs) show how well a mother wavelet function correlates with the signal. If the signal has a major component of frequency corresponding to the current scale, then the wavelet at this scale will be similar to the signal at the particular location, where this frequency component occurs. Consequently, CWC will have a large value at this location and scale. Therefore, in this research CWC-SVS have been determined using 324 mother wavelet functions from the following families: Haar, Daubechies, Symlet, Coiflet, Gaussian, Morlet, complex Morlet, Mexican hat, bio-orthogonal, reverse bio-orthogonal, Meyer, discrete approximation of Meyer, complex Gaussian, Shannon, and frequency B-spline. To find the best mother wavelet, central moments of wavelet coefficients were deliberated in this research.

Computing the central moments [27] are similar to those performed to determine mechanical moments, such as the moment of inertia of an object. The term "central" refers that the various statistical moments are computed with consideration to the centroid of the probability density function of the population. The mth central moment is as follows:

$$\left\langle (x-x')^m \right\rangle = E\left[(x-x')^m\right] = \mu_m \int_{-\infty}^{+\infty} (x-x')^m \, p(x)dx \quad (4)$$

The $\langle\ \rangle$ or $[\ ]$ means the expected value. This is the value that is expected (in the probabilistic sense) if the integral is performed. When the centroid or mean, x', equals to 0, the central moments can be defined as moments about the origin:

$$\left\langle (x)^m \right\rangle = \mu_m \int_{-\infty}^{+\infty} x^m \, p(x)\,dx \quad (5)$$

Further, the central moment can be related to the moment about the origin by the following transformation:

$$\mu_m = \sum (-1)^i \binom{m}{i} \mu_1^i \, \mu'_{m-i} \quad (6)$$

The zeroth central moment, μ_0, is an identity equal to 1:

$$\mu_0 = \int_{-\infty}^{+\infty} p(x)\,dx = 1 \quad (7)$$

The first central moment is the definition of the mean value and the second one, μ_2, defined the variance, σ^2, as:

$$\mu_2 = \sigma^2 = \int_{-\infty}^{+\infty} (x - x')^2 \, p(x)\,dx \quad (8)$$

The standard deviation, σ, is the square root of the variance which describes the width of the probability density function. The variance of x can be expressed in terms of the expectation of x^2, $E[x^2]$, and the square of the centroid of x, x'^2 and

therefore the Equation (4) for this case becomes:

$$\sigma^2 = E[x^2] - x'^2 \quad (9)$$

The third central moment, μ_3, is used in the definition of the skewness, Sk:

$$Sk = \frac{\mu_3}{\sigma^3} = \frac{1}{\sigma^3} \int_{-\infty}^{+\infty} (x - x')^3 \, p(x)\,dx \quad (10)$$

Skewness explains the symmetry of the probability density function, where a positive skew indicates a distribution stretched to the right.

The fourth central moment, μ_4, is used in the definition of the kurtosis, Ku, where

$$Ku = \frac{\mu_4}{\sigma^4} = \frac{1}{\sigma^4} \int_{-\infty}^{+\infty} (x - x')^4 \, p(x)\,dx \quad (11)$$

has no units and it explains the peakedness of the probability density function.

To discriminate the best mother wavelet 4×50 sample signals for three faulty and one normal condition were considered (see Figures 3 to 6). Amongst these signal characteristics (standard deviation, variance, kurtosis, 4th central moment),4th central moment and kurtosis appear to have the highest discriminability for broken-tooth gear. Since the amplitude of 4th central moment for broken-tooth gear is extremely high, the discrimination for other conditions is not clear in the figure. However, 4th central moment has been proven as the best feature for discriminating all defects in this research.

In sum, the best mother function has been selected based on the following steps:

1. The average of feature vector (variance of CWC-SVS, including 2^4 numbers per sample), was computed in 50 signals segments for each condition.
2. Variance of the mentioned average was determined for 2^4 elements, and the 5 highest elements of the vector have been selected as the best features to maximize variance increasing the difference between faults.
3. Finally, the sum of 5 elements, “SUMVAR” criterion, was

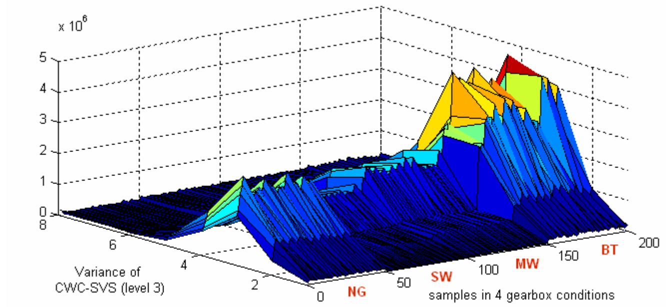

Fig.3. Variance of CWC-SVS with db44 in level 3 (8 features)

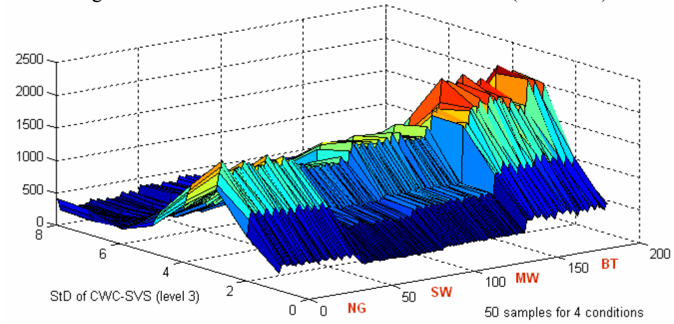

Fig. 4. Standard Deviation of CWC-SVS with db44 in level 3 (8 features)

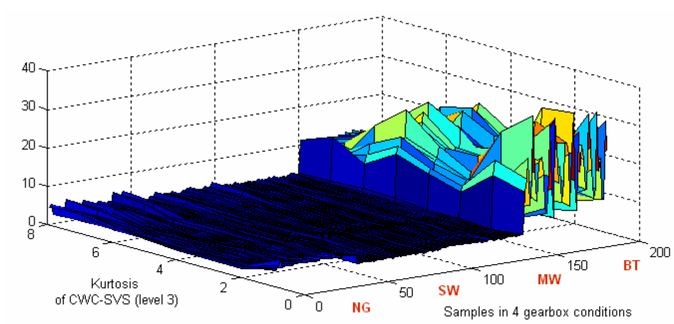

Fig. 5. Kurtosis of CWC-SVS with db44 in level 3 (8 features)

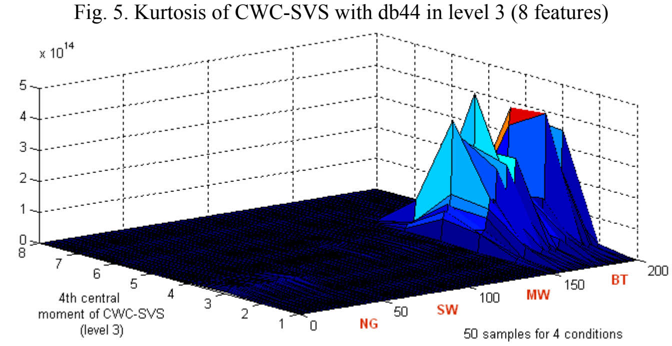

Fig. 6. 4th central moment of CWC-SVS with db44 in level 3 (8 features)

applied to compare 324 candidate mother functions. A higher SUMVAR indicates a higher correlation between mother wavelet and signals and a greater difference between fault types. db 44 has a fit hundreds of times greater than the other mother wavelets (see Figure 7). This is remarkable because the difference between two mother wavelet functions in db family with adjacent order (such as db5 and db6) is slight, but the lower order wavelets are very similar to each other. In high order db, the distinction between the adjacent orders is very different (see Figure 8). The shape of db 44 is near symmetric and that would be one point the other high order db lack. As shown in figure 7, db44 and db45 are the sole clear-cut choice for gearbox monitoring and the others perform poorly in comparison.

After db44-45, bior 3.1 and some complex mother wavelets have higher similarity to the signals (see Figure 9). Morlet is the most common function in machine condition monitoring and it has been extensively applied in previous research; consistent with previous work, the SUMVAR criterion indicates that Morlet is an appropriate function as compared with many other functions, such as Daubechies (1 to 43), Coiflet, Symlet, complex Morlet, Gaussian, complex Gaussian, and Meyer (see Figure 10).

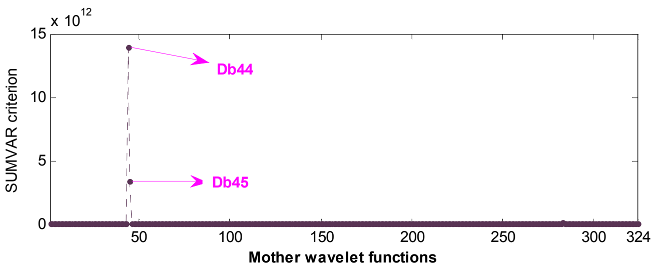

Fig.7. SUMVAR vs. 324 mother wavelets

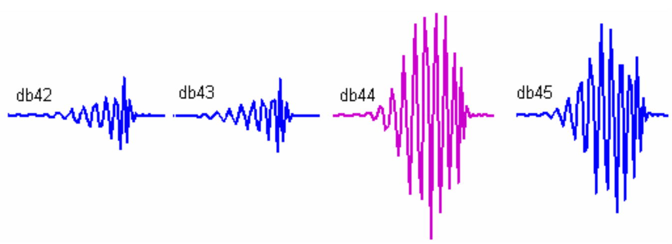

Fig. 8. High order Daubechies

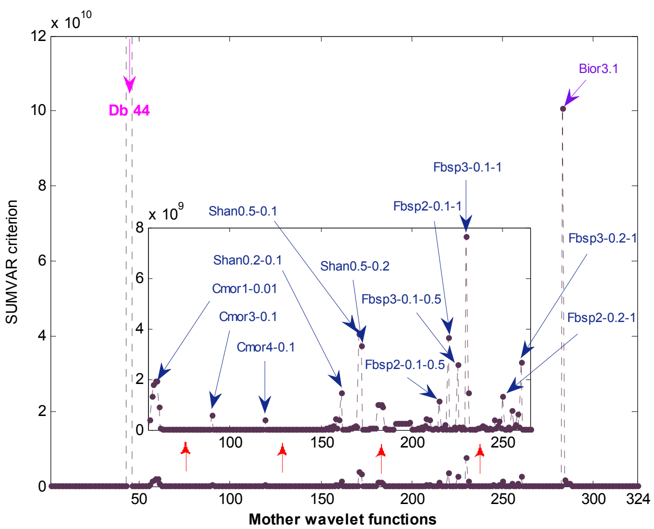

Fig. 9. Trend of complex mother wavelets vs. SUMVAR

C. *Results and Discussion*

The result shows that among 324 mother wavelets, which have been explained in Table 1 and 2 in detail, db44 acts very distinctly as CWC-SVS. For example, db44 and Haar are compared directly in a sample signal of broken-tooth gear in Figure 11. In db44, the magnitude of coefficients is unique and hundreds of time more than the others. It is obvious that db 44 has the proper attribute to match with gear vibration signals. The one clear drawback for high-order Daubechies is that they require more computation time. Figure12 shows the computational time of CWC-SVS for different wavelet functions in a sample signal by a Pentium 4 PC.

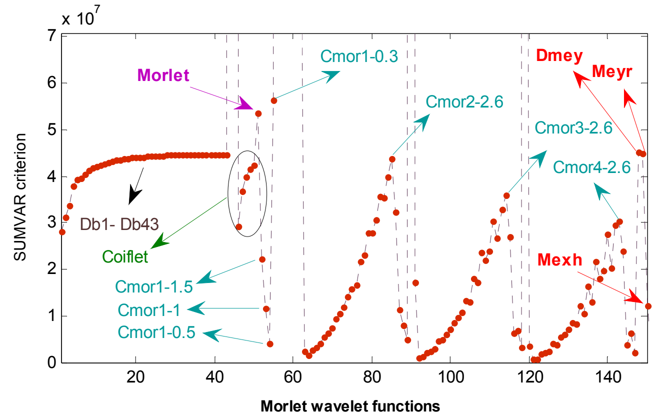

Fig. 10. The trend of other mother wavelets vs. SUMVAR

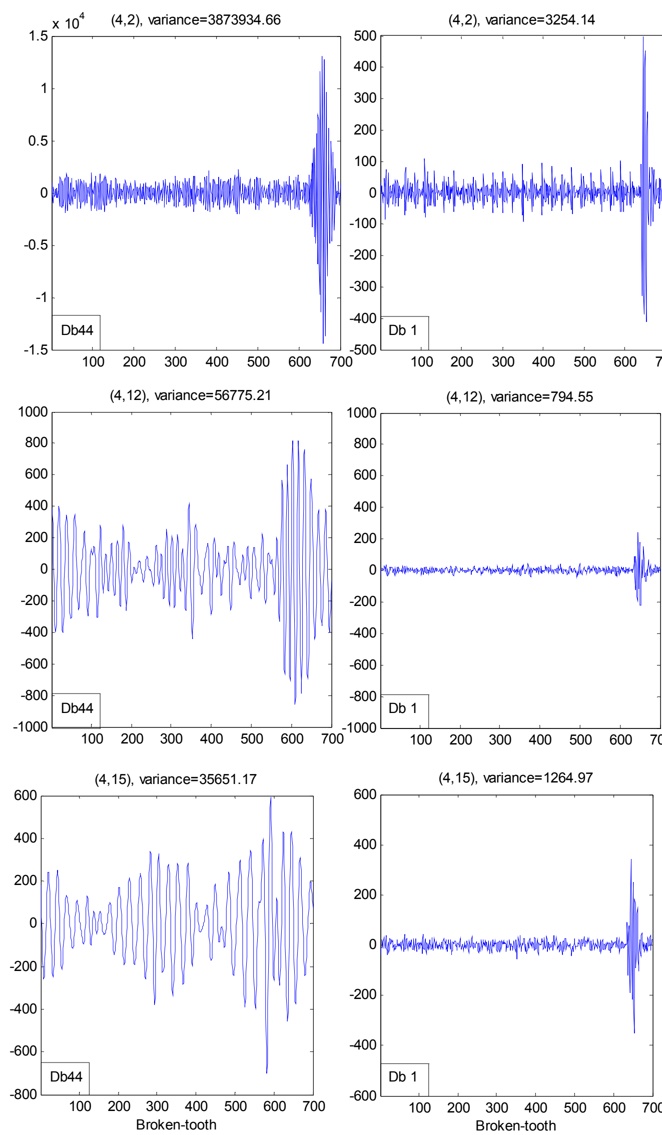

Fig. 11: CWC-SVS of a sample signal of Broken-tooth

TABLE 1. STUDIED WAVELET FAMILIES IN THIS RESEARCH

No.	Family (short form)	Order	Best order
1	Haar (db1)	db 1	Db 1
2-45	Daubechies(db)	db 2 to 45	Db 44
46-50	Coiflet (coif)	coif 1 to 5	Coif 5
51	Morlet (Morl)	morl	Morl
52-147	Complex Morlet (cmor Fb-Fc)*	table 2	Cmor 1-0.1
148	Discrete Meyer (dmey)	dmey	Dmey
149	Meyer (meyr)	meyr	Meyr
150	Mexican Hat (mexh)	mexh	Mexh
151-200	Shannon (Shan Fb-Fc)*	table 2	Shan 1-0.1
201-260	Frequency B-Spline (fbsp M-Fb- Fc)*	table 2	Fbsp 2-1-0.1
261-267	Gaussian (gaus)	gaus 1 to gaus7	Gaus 7
268-275	Complex Gaussian (cgau)	cgau 1 to cgau8	Cgau 8
276-290	Biorthogonal (bior Nr.Nd)**	table 2	Bior 3.1
291-305	Reverse Biorthogonal (rbio Nr.Nd)**	table 2	Rbio 5.5
306-324	Symlet (sym)	sym 2 to 20	Sym 15

* Fb is a bandwidth parameter, Fc is a wavelet center frequency, M is an integer order parameter

** Nr and Nd are orders: r for reconstruction / d for decomposition

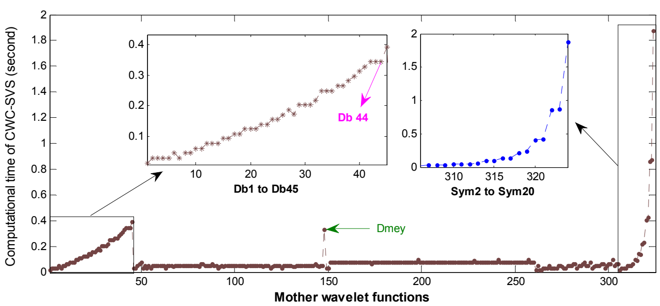

Fig. 12. Computational time of CWC-SVS of a sample signal (by a P4 PC)

III. CONCLUSION

To summerize in short:

1. db44 possesses the most similarity to faulty gears. It matches with randomly high impacts as well. The ability of db44 to work across idiosyncratic signals allows the development of other methods based on the resemblance of the signal and mother wavelet in condition monitoring.

2. The 4th central moment of CWC is a new identifying feature for different kinds of the defects, particularly uncorrelated random excitations such as broken-tooth gear. Standard deviation and variance of CWC also were supported as discriminating features.

3. The statistical criteria, including standard deviation, variance, kurtosis, and 4^{th} central moments of CWC, can be used to train intelligent systems (such as ANNs), because high fluctuations across different small-size samples of signalsmake them highly discriminable by this method.

4. Piecewise Cubic Spline Interpolation synchronized the vibration signals with different lengths exceptionally well in this study. This permitted the use of symmetric mother wavelets, which have superior results in such vibration signals. Although db family is not symmetric, db44 possesses the near symmetric attribute as depicted in figure 8.

TABLE 2. STUDIED WAVELET FAMILIES IN DETAIL

No	Wave	No	Wave	No	Wave	No	Wave
52	1-1.5	100	3-1.1	148	dmey	196	2-0.6
53	1-1	101	3-1.2	149	meyr	197	2-0.7
54	1-0.5	102	3-1.3	150	mexh	198	2-0.8
55	1-0.3	103	3-1.4	151	0.1-0.1	199	2-0.9
56	1-0.2	104	3-1.5	152	0.1-0.2	200	1-1
57	1-0.1	105	3-1.6	153	0.1-0.3	201	1-0.1-0.1
58	1-0.05	106	3-1.8	154	0.1-0.4	202	1-0.1-0.2
59	1-0.02	107	3-1.9	155	0.1-0.5	203	1-0.1-0.3
60	1-0.01	108	3-2	156	0.1-0.6	204	1-0.1-0.4
61	2-0.1	109	3-2.1	157	0.1-0.7	205	1-0.1-0.5
62	2-0.2	110	3-2.2	158	0.1-0.8	206	1-0.1-0.6
63	2-0.3	111	3-2.3	159	0.1-0.9	207	1-0.1-0.7
64	2-0.4	112	3-2.4	160	0.1-1	208	1-0.1-0.8
65	2-0.5	113	3-2.5	161	0.2-0.1	209	1-0.1-0.9
66	2-0.6	114	3-2.6	162	0.2-0.2	210	1-0.1-1
67	2-0.7	115	3-2.7	163	0.2-0.3	211	2-0.1-0.1
68	2-0.8	116	3-2.8	164	0.2-0.4	212	2-0.1-0.2
69	2-0.9	117	3-2.9	165	0.2-0.5	213	2-0.1-0.3
70	2-1	118	3-3	166	0.2-0.6	214	2-0.1-0.4
71	2-1.1	119	4-0.1	167	0.2-0.7	215	2-0.1-0.5
72	2-1.2	120	4-0.2	168	0.2-0.8	216	2-0.1-0.6
73	2-1.3	121	4-0.3	169	0.2-0.9	217	2-0.1-0.7
74	2-1.4	122	4-0.4	170	0.2-1	218	2-0.1-0.8
75	2-1.5	123	4-0.5	171	0.5-0.1	219	2-0.1-0.9
76	2-1.6	124	4-0.6	172	0.5-0.2	220	2-0.1-1
77	2-1.8	125	4-0.7	173	0.5-0.3	221	3-0.1-0.1
78	2-1.9	126	4-0.8	174	0.5-0.4	222	3-0.1-0.2
79	2-2	127	4-0.9	175	0.5-0.5	223	3-0.1-0.3
80	2-2.1	128	4-1	176	0.5-0.6	224	3-0.1-0.4
81	2-2.2	129	4-1.1	177	0.5-0.7	225	3-0.1-0.5
82	2-2.3	130	4-1.2	178	0.5-0.8	226	3-0.1-0.6
83	2-2.4	131	4-1.3	179	0.5-0.9	227	3-0.1-0.7
84	2-2.5	132	4-1.4	180	0.5-1	228	3-0.1-0.8
85	2-2.6	133	4-1.5	181	1-0.1	229	3-0.1-0.9
86	2-2.7	134	4-1.6	182	1-0.2	230	3-0.1-1
87	2-2.8	135	4-1.8	183	1-0.3	231	1-0.2-0.1
88	2-2.9	136	4-1.9	184	1-0.4	232	1-0.2-0.2
89	2-3	137	4-2	185	1-0.5	233	1-0.2-0.3
90	3-0.1	138	4-2.1	186	1-0.6	234	1-0.2-0.4
91	3-0.2	139	4-2.2	187	1-0.7	235	1-0.2-0.5
92	3-0.3	140	4-2.3	188	1-0.8	236	1-0.2-0.6
93	3-0.4	141	4-2.4	189	1-0.9	237	1-0.2-0.7
94	3-0.5	142	4-2.5	190	1-1	238	1-0.2-0.8
95	3-0.6	143	4-2.6	191	2-0.1	239	1-0.2-0.9
96	3-0.7	144	4-2.7	192	2-0.2	240	1-0.2-1
97	3-0.8	145	4-2.8	193	2-0.3	241	2-0.2-0.1
98	3-0.9	146	4-2.9	194	2-0.4	242	2-0.2-0.2
99	3-1	147	4-3	195	2-0.5	243	2-0.2-0.3

5. The center frequency should be considered for different case studies in complex mother functions. For example, complex mother wavelets with a center frequency of 0.1 resulted in better identification of faulty gears.

6. The paper also supports prior research in machine condition monitoring identifying the Morlet wavelet function as the best within this field. In comparison to the other wavelets frequently applied in signal analysis (e.g. Daubechies, Coiflet, Symlet, complex Morlet, Gaussian, complex Gaussian, Meyer, etc.), Morlet possesses the better fit based on SUMVAR criteria (see Figure 10).

8. The authors think, among mother wavelet candidates, db44 would be superior across gears, bearings, and other types of similar vibration signals. db44 offers a method for

distinguishing the similar signals. Future research concerning mother wavelet functions might focus on optimizing db44 [28] for such purposes.

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