# Reduced Complexity Detection for BICM MIMO OFDM System

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***Abstract*—This paper presents a new approach to the soft demodulation for spatial data streams in MIMO channels in slow fading channels. We focus on high spectral efficiency bit interleaved coded modulation (BICM) MIMO OFDM system where, after serial to parallel conversion and per antenna coding, spatial data streams are simultaneously transmitted by using an antenna array. We propose a detection algorithm basing on a combination of linear and non linear detection. The proposed algorithm orders the spatial streams as per V-BLAST ordering and then uses linear detectors as MMSE or zero forcing (ZF) to detect the streams which have seen good channel realizations and therefore enjoy higher signal-to-noise-ratio (SNR). These streams after being detected are subsequently stripped off leading to the max log MAP detection of the streams which have seen comparatively poor realizations of the channel and consequently have lower SNR. Unlike the maximum likelihood (ML) detection, whose complexity is huge due to the need for enumerating all possible combinations of transmitted constellation points, the proposed method has low complexity. The algorithm has a fully parallel structure, suitable for the implementation in parallel hardware. Numerical examples illustrate its performance on slow fading $3 \times 3$ and $4 \times 4$ complex MIMO channels.***

## I. Introduction

The seminal works in [1] and [2] on multiple antenna elements at the transmitter and receiver show a huge increase in the throughput of this point-to-point channel, referred to also as multiple input multiple output (MIMO) system. These promising results shifted the focus of research on multi antenna communications and motivated the introduction of multiple antenna elements in the future communication systems as IEEE 802.11n [3], IEEE 802.16m [4] and 3GPP LTE [5]. Enhanced capacity (degrees of freedom) and improved reliability (diversity) are the two significant gains of MIMO communications which need to be traded off [6].

Researchers persist to strive for finding space time codes (STC) with reduced decoding complexity. These codes take into account both the spatial and temporal dimensions of the MIMO channel. Orthogonal Space-Time Block Codes (OSTBCs) [7] are widely used because they are easy to encode and decode. OSTBCs are repetition codes that only provide diversity gain. In order to approach the capacity limit they have to be used in concatenation with an outer code. Remarkable coding gains can be obtained if a capacity achieving temporal encoder, such as turbo or Low-Density Parity Check (LDPC) is used in concatenation with a STC [8]. Recently it has been shown for the ergodic channels that the complex concatenation of space time code and outer codes can be replaced with coded spatial streams for achieving capacity [9]. Each spatial stream is independently coded using temporal encoders as convolutional, turbo or LDPC codes where at the receiver, standard off-the-shelf decoders are used after the demodulator.

After appropriate filtering and sampling, the well-known data model is $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$ (to be made precise in Section II) where $\mathbf{y}$ is the received data, $\mathbf{H}$ is the channel matrix, $\mathbf{x}$ is the symbol vector with the elements from a finite constellation, and $\mathbf{z}$ is the noise. The problem is then to detect $\mathbf{x}$ from $\mathbf{y}$. Essentially the same problem occurs in multiuser detection for CDMA [10] and for single-carrier transmission over channels that induce intersymbol interference. In these cases, the matrix $\mathbf{H}$ usually has a specific structure. We are interested in the fundamental aspects of the model $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$ and we shall refer to it in general terms as a MIMO model.

The problem of detecting $\mathbf{x}$ from $\mathbf{y}$ has stimulated a large body of research [10]. One can easily show that if the noise $\mathbf{z}$ is Gaussian then obtaining the maximum-likelihood solution is equivalent to minimizing the Euclidean distance $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$ with respect to $\mathbf{x}$ over the finite set spanned by all possible combinations of constellation points that can constitute the vector $\mathbf{x}$. Unfortunately this problem is NP-hard for general $\mathbf{H}$ and $\mathbf{y}$ [11] which implies that there are no known efficient (i.e. polynomial-time) solutions. Many sophisticated methods as lattice reduction exist which find the maximum likelihood (ML) solution with high probability, but these methods are in general computationally very complex. This is true also in an average sense if $\mathbf{H}$ is random (i.e. for a fading channel). The popular sphere decoding method [12], for example, is much more efficient than a brute-force search, but it still admits an average complexity that is exponential in the dimension of $\mathbf{x}$.

Naive solutions, like neglecting the integer constraint and then projecting the so-obtained solution onto the finite set of permissible $\mathbf{x}$ (linear receivers), in general works poorly except if $\mathbf{H}$ is well conditioned. One can do somewhat better by using decision-feedback-equalization detection (nulling-and-canceling), whereby the elements of $\mathbf{x}$ are detected one by one, and in an order that can be optimally chosen [13]. Linear receivers as MMSE or zero forcing (ZF) have lower complexity as compared to the brute-force search but exhibit degraded performance especially at lower SNRs. Different spatial streams in a MIMO system experience different SNRs as each stream sees a different realization of the channel and

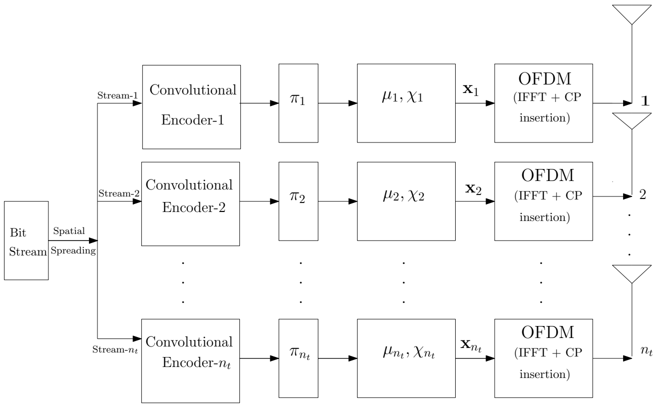


Fig. 1. Block diagram of the transmitter of $n_t \times n_r$ BICM MIMO OFDM system. $\pi_1$ denotes the random interleaver, $\mu_1$ the labeling map, $\chi_1$ the signal set and $\mathbf{x}_1$ the complex symbols vector of stream-1.

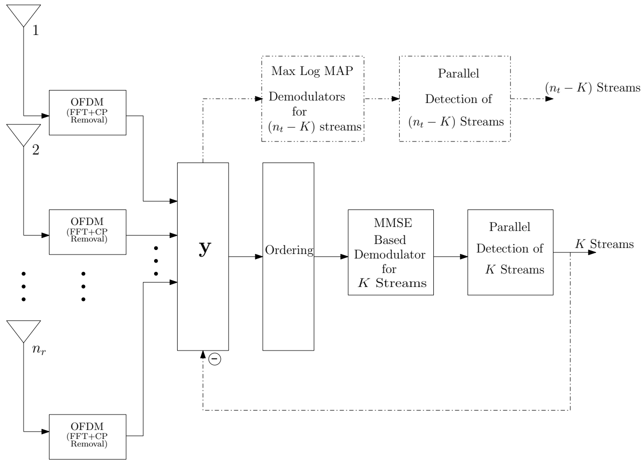


Fig. 2. Block diagram of the receiver of BICM MIMO OFDM system.

the spatial streams with the relatively lower SNR will dominate the error performance of the system.

We propose a detection algorithm in which the spatial streams which have experienced higher SNR are detected using linear detectors as ZF or MMSE and are subsequently stripped off while the spatial streams which have experienced lower SNR are detected using max log MAP algorithm. The algorithm therefore successfully escapes the exponential complexity of MIMO detection and the degraded performance of linear receivers. This leads to a performance (error rate) better than the standard linear MMSE based approaches and comparable to that of List Sphere detection (LSD) [14] coupled with a significant reduction in the complexity. Our proposed algorithm can also be extended to equal-rate non-uniform power distribution between these spatial streams in view of successive stripping [15].

We consider in this paper bit interleaved coded modulation (BICM) MIMO OFDM system as it provides a promising choice for next-generation wireless networks where MIMO enhances the spectral efficiency, OFDM reduces the complexity of equalization and BICM [16] stands as a robust coding scheme for fading channels. Regarding notations, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors and uppercase boldface letters for matrices. $|.|$ and $\|.\|$ indicate norm of scalar and vector respectively while $(.)^T$ and $(.)^\dagger$ indicate transpose and conjugate transpose respectively. The paper is organized as follows. In section II we provide the system model while section III gives a review of the existing approaches. In section IV we propose low complexity detection algorithm and section V describes simulation results which is followed by the conclusions.

## II. System Model

We consider a MIMO system which is a $n_t \times n_r$ $(n_r \geq n_t)$ BICM MIMO OFDM system with $n_t$ spatial streams. The detection is based on segregating the streams into two groups based on their SNR. It is followed by the decoding of the first group of streams which is subsequently stripped off leading to the detection of the second group of streams. Block diagrams of the transmitter and receiver are shown in the figures 1 and 2 respectively. Well known baseband model of the system at the $n$-th frequency tone is given as:-

$$\mathbf{y}_n = \mathbf{h}_{1,n}x_1 + \mathbf{h}_{2,n}x_2 + \cdots + \mathbf{h}_{n_t,n}x_{n_t} + \mathbf{z}_n, \quad n = 1, 2, \cdots, N$$

where $N$ is the total number of frequency tones and $n_t$ is the total number of spatial streams/transmit antennas. We can conveniently drop the frequency index and can rewrite the system equation as

$$\begin{aligned} \mathbf{y} &= \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \cdots + \mathbf{h}_{n_t} x_{n_t} + \mathbf{z} \\ \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{z} \end{aligned} \tag{1}$$

where $\mathbf{y}, \mathbf{z} \in \mathbb{C}^{n_r}$ are the vectors of the received symbols and circularly symmetric complex white Gaussian noise of variance $N_0$ at the $n_r$ receive antennas. $\mathbf{h}_k \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from the $k$-th transmitting antenna to $n_r$ receive antennas and $x_k$ is the complex symbol of the $k$-th stream transmitted by the $k$-th transmit antenna with $E\left[|x_k|^2\right] = \sigma_k^2$. It is assumed that each channel path between the transmitter and the receiver is independent. The complex symbols $x_1, \cdots, x_{n_t}$ of $n_t$ streams are also assumed independent.

$\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \cdots \mathbf{h}_{n_r}]$ is the channel matrix with complex channel gains $E\left[|h_{ij}|^2\right] = 1$. Under the power constraint $P_T$, the average SNR at each receiver branch is $\frac{P_T}{N_0}$.

## III. Review of Existing Schemes

We now review some existing schemes for MIMO detection.

*1) MMSE:* Detection based on MMSE equalization [14] involves linear MMSE preprocessing i.e. applying a spatial filter $\mathbf{M}$ to the received signal vector $\mathbf{y}$ i.e. $\tilde{\mathbf{x}} = \mathbf{M}\mathbf{y}$ where $\tilde{\mathbf{x}}$ is the biased estimate of $\mathbf{x} = [x_1, x_2, \cdots, x_{n_t}]^T$. $\mathbf{M}$ is given as

$$\mathbf{M} = \left(N_0 \mathbf{P}^{-1} + \mathbf{H}^\dagger \mathbf{H}\right)^{-1} \mathbf{H}^\dagger$$

where $\mathbf{P}$ is the diagonal power distribution matrix with the diagonal as $\left[\sigma_1^2, \sigma_2^2, \cdots, \sigma_{n_t}^2\right]$. It is followed by an unbiasing operation i.e. $\hat{\mathbf{x}} = \Gamma^{-1}\tilde{\mathbf{x}}$ where $\Gamma = \text{diag}\left(\mathbf{MH}\right)$. Based on the Gaussian assumption of post detection interference, MMSE preprocessing decouples the spatial streams and the bit metric for the $i$-th bit for bit value $b$ of the symbol $x_k$ on $k$-th stream is given as

$$\lambda_k^i\left(\mathbf{y}, b\right) \approx \max_{x_k \in \chi_{k,b}^i} \left[-\frac{\gamma_k^2}{N_0}\left|\hat{x_k} - x_k\right|^2\right] \tag{2}$$

for $k = 1, 2, \cdots, n_t$ where $\gamma_k$ is the $k$-th diagonal element of $\Gamma$. $\chi_{k,b}^i$ denotes the subset of the signal set $x_k \in \chi_k$ whose labels have the value $b \in \{0, 1\}$ in the position $i$.

*2) Max Log MAP:* In a $n_t \times n_r$ system, the ML bit metric for bit $b$ at the $i$-th location of the $k$-th stream $x_k$ is given as

$$\lambda_k^i(\mathbf{y}, b) = \log \sum_{x_1 \in \chi_1} \cdots \sum_{x_k \in \chi_{k,b}^i} \cdots \sum_{x_{n_t} \in \chi_{n_t}} \exp\left[-\frac{1}{N_0}\|\mathbf{y} - \mathbf{Hx}\|^2\right]$$

Applying log sum approximation [16] we have

$$\lambda_k^i\left(\mathbf{y}, b\right) \approx \max_{x_1 \in \chi_1 \cdots x_k \in \chi_{k,b}^i \cdots x_{n_t} \in \chi_{n_t}} \left[-\frac{1}{N_0}\left\|\mathbf{y} - \mathbf{Hx}\right\|^2\right]$$

Finding the maximum term in this sum is equivalent to minimizing $\|\mathbf{y} - \mathbf{Hx}\|^2$ subject to the constellation constraint on $\mathbf{x}$ , and this is an NP-hard optimization problem. Therefore the max-log approximation, while it is conceptually simple, still suffers from being computationally very intensive. Minimizing $\|\mathbf{y} - \mathbf{Hx}\|^2$ by hard decision sphere decoding [12] is feasible, but incurs exponential average complexity. This limits the range of problem sizes which may be addressed. Moreover, the complexity of this approach is random (i.e. it depends on the realizations of $\mathbf{H}$ and $\mathbf{x}$ ) which may introduce random decoding delays. Additionally sphere decoding is based on searching through a tree in a sequential manner and therefore is difficult to execute efficiently on parallel hardware architectures.

## IV. Proposed Detection Algorithm

Proposed algorithm exploits the low complexity of linear receivers and the improved performance of max log MAP detectors. The fact that the linear receivers as MMSE have significantly lower complexity as compared to the brute-force search but exhibit degraded performance particularly at lower SNRs is exploited by detecting the spatial streams which have seen good realizations of the channel and therefore enjoy relatively higher SNR using these linear filters. These detected streams are subsequently stripped off leading to a group of yet undetected streams which have seen comparatively poor realizations of the channel and therefore have relatively lower SNR. This group of streams is then detected using max log MAP demodulators. So the proposed detection algorithm can be divided into 3 stages.

### *A. Optimal Detection Ordering*

The system performance is affected by the order in which the spatial streams are detected. We consider the V-BLAST criteria by simply ordering the streams as per the decreasing post detection SNR. This approach is termed as the ”best first” cancellation approach within the multi-user community. The spatial streams to be detected by MMSE filter are decided by choosing the rows of $\mathbf{M}$ with the smallest norms. Depending on the system parameters, a threshold for the row norm of MMSE filter $\mathbf{M}$ can be decided as a criteria for the detection using MMSE filters.

### *B. Detection based on linear MMSE filters*

Once the spatial stream to be detected by MMSE filter are decided, these streams are detected using MMSE filters. The MMSE filter for detecting $k$-th stream which is the $k$-th row of $\mathbf{M}$ is given as

$$\mathbf{m}_k = \left(\mathbf{h}_k^\dagger \mathbf{R}_k^{-1} \mathbf{h}_k + 1/\sigma_k^2\right)^{-1} \mathbf{h}_k^\dagger \mathbf{R}_k^{-1} \tag{3}$$

where

$$\mathbf{R}_k = \sigma_1^2 \mathbf{h}_1 \mathbf{h}_1^\dagger + \cdots + \sigma_{k-1}^2 \mathbf{h}_{k-1} \mathbf{h}_{k-1}^\dagger + \sigma_{k+1}^2 \mathbf{h}_{k+1} \mathbf{h}_{k+1}^\dagger + \cdots + \sigma_{n_t}^2 \mathbf{h}_{n_t} \mathbf{h}_{n_t}^\dagger \tag{4}$$

Application of MMSE filter on received vector $\mathbf{y}$ yields

$$\mathbf{m}_k \mathbf{y} = y_k = \alpha_k x_k + z_k \tag{5}$$

So the bit metric for bit $b$ at $i$-th location of $k$-th spatial stream is given as

$$\lambda_k^i\left(\mathbf{y}, b\right) \approx \max_{x_k \in \chi_{k,b}^i} \left[-\frac{1}{N_k}\left|y_k - \alpha_k x_k\right|^2\right] \tag{6}$$

where $N_k = \mathbf{m}_k \mathbf{R}_k \mathbf{m}_k^\dagger$. Without loss of generality, let the last $K$ spatial streams are detected using MMSE filters which are subsequently stripped off.

### *C. Max Log MAP*

For the remaining $n_t - K$ spatial streams which have experienced poor channel realizations, max log MAP detector is used. The new system equation is

$$\mathbf{y}' = \mathbf{H}'\mathbf{x}' + \mathbf{z}' \tag{7}$$

where $\mathbf{y}', \mathbf{z}' \in \mathbb{C}^{n_r}$ and $\mathbf{H}'$ is $n_r \times (n_t - K)$ complex matrix and $\mathbf{x}' \in \mathbb{C}^{n_t - K}$. The bit metric for bit $b$ at the $i$-th location of the $k$-th stream $x_k$ is given as

$$\lambda_k^i\left(\mathbf{y}, b\right) \approx \max_{x_1 \in \chi_1 \cdots x_k \in \chi_{k,b}^i \cdots x_{n_t - K} \in \chi_{n_t - K}} \left[-\frac{1}{N_0}\left\|\mathbf{y}' - \mathbf{H}'\mathbf{x}'\right\|^2\right]$$

The computational complexity of this detection process is $\mathcal{O}(|\chi|^{n_t - K})$. Closer the $K$ is to $n_t$, lower is the complexity of detection.

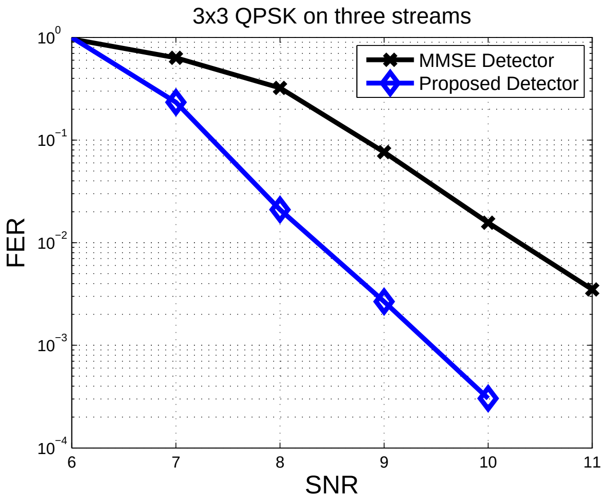


Fig. 3. $3 \times 3$ MIMO system with QPSK on three spatial streams.

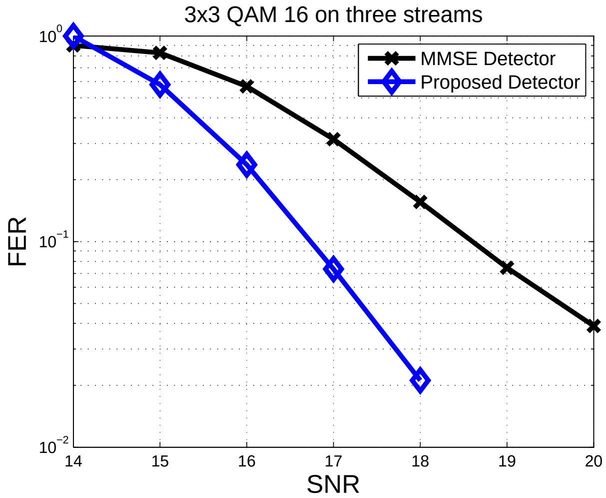


Fig. 4. $3 \times 3$ MIMO system with QAM16 on three spatial streams.

## V. Simulation Results

We now assess the performance of the proposed demodulator by means of simulation of the frame-error rates (FER). We focus on target error-rates of the order of $10^{-2}$. We consider $3 \times 3$ and $4 \times 4$ BICM MIMO OFDM systems using the *de facto* standard, $64$ state, rate-$1/2$ convolutional encoder. The upcoming WLAN standard 802.11n [3] supports the codeword sizes of 648, 1296, and 1944 bits. For our purposes, we selected the codeword size of 1296 bits. The MIMO channel has iid Gaussian matrix entries with unit variance. The channel is independently generated for each frame and perfect CSI at the receiver is assumed. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider the MMSE based standard approach and the proposed approach. Spatial streams of equal rates and equal power are transmitted in a $3 \times 3$ and $4 \times 4$ system. Figures 3, 4 5 and 6 indicate much improved performance of the proposed approach with respect to that of MMSE based approach in $3 \times 3$ and $4 \times 4$ system.

## VI. Conclusions

We have presented a novel detection scheme for BICM MIMO OFDM system. Our proposed detector is based on the combination of linear and non linear detectors and has reduced computational complexity as compared to the brute force search and better performance than the standard linear equalizer based solutions as MMSE filter. The proposed detector exploits the low complexity of linear detectors and better performance of max log MAP detectors. We have used convolutional codes because of its widespread application in existing MIMO systems as IEEE 802.11n [3] but future work shall examine the influence of more powerful turbo codes on the proposed detector.

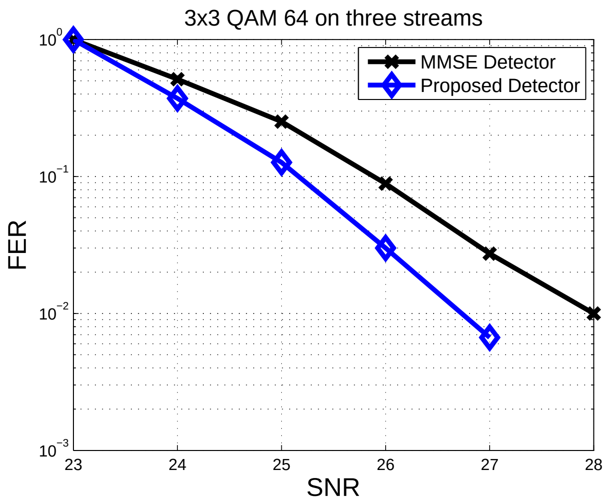


Fig. 5. $3 \times 3$ MIMO system with QAM64 on three spatial streams.

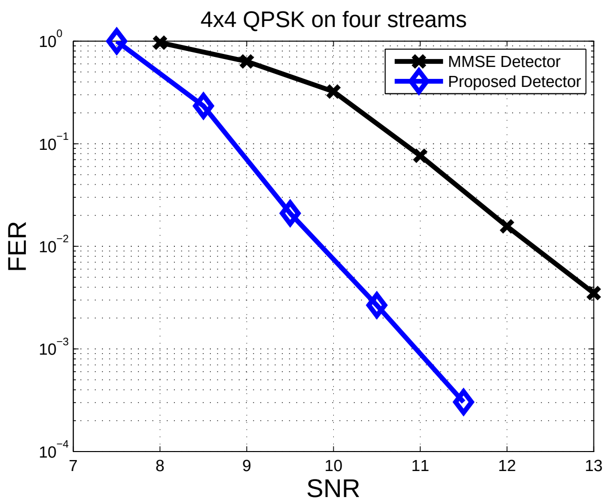


Fig. 6. $4 \times 4$ MIMO system with QPSK on four spatial streams.

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