

# Extracting Full Dynamic Equations of 3 Degree of Freedom Rotary Table

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**Abstract**— A 3 Degree of Freedom Rotary Table is a robotic mechanism which brings about rotational movements. This mechanism helps to make a movement scenario in order to test Guidance and Navigation Systems of moving vehicles. It has complicated dynamic equations. Having these equations in hand is crucial for control objectives. It is shown that we have obtained these equations by applying the Newton's Second Law for Rotation instead of using common methods in robotic modeling such as Denavit-Hartenberg and Newton-Euler methods. We have extracted full dynamic equations that contain all designing parameters such as Moment of Inertia factor in all round and even the unbalanced and wobble effect. Finally we have verified obtained equations by simulating them in MATLAB software.

## I. INTRODUCTION

A 3 Degree of Freedom Rotary Table is a mechanism which has 3 rotary gimbals, called: outer gimbal, middle gimbal and inner gimbal. As shown in either Fig. 1 or 2, these gimbals rotate perpendicular in proportion to each other. The U.U.T, which is the abbreviation of the Unit under Test, is fixed to inner gimbal and rotates with it. In fact, we rotate gimbals in order that the U.U.T is rotated.

Reviewing some paper which published about rotary tables during past years resulted in the followings: (1) these papers have been focused on 1 or 2 Degree of Freedom Rotary Tables more (2) there was not any detail study in modeling of a 3 Degree of Freedom Rotary Table.

This paper will be organized as follows; second section will discuss preliminary definitions and notations. The third one discusses the kinematics of 3 Degree of Freedom Rotary Table. Forth and fifth section discuss the dynamics and the sixth section verifies the calculated equations

## II. PRELIMINARIES

Here is the main formula; we apply to reach 3 Degree of Freedom Rotary Table equations: Newton's Second Law for Rotation.

$$M_{OK} = P_i(H_{OK}), H_{OK} = J_{OK}\omega_{i-OK} \quad (1)$$

Equation 1 is applied for every point; each point is shown with  $k$  symbol.  $M_{OK}$  is the external torques round, enforced

to the  $k$  point.  $H_{OK}$  is the rotational momentum of the  $k$  point in proportion to the inertia point, shown as  $O$ .  $J_{OK}$  is the  $k$  point moment of inertia in proportion to the inertia point.  $\omega_{i-OK}$  is the rotational velocity of  $k$  point in proportion to the inertia point which is defined in the inertia coordinates frame. Finally  $P_i(\cdot)$  is differential of something in the inertia coordinates frame.

If we calculate the Equation 1 for whole  $k$  points in proportion to another point instead of the inertia point, we will reach to the Equation 2.

$$\sum_K M_{AK} = P_i \left( \sum_K H_{AK} \right) + \left( \sum_K m_K r_{AK} \right) \times P_i^2 r_{OA} \quad (2)$$

Where

$${}^B P_i \left( \sum_K H_{AK} \right) = P \left( \sum_K H_{AK} \right) + \left( \omega_{iB} \times \left( \sum_K H_{AK} \right) \right) \quad (3)$$

And

$$\left( \sum_K H_{AK} \right) = {}^B J_A \cdot {}^B \omega_{iB} \quad (4)$$

Applying the Equation 3 and 4 in the Equation 2, resulted in Equation 5.

$${}^B M_A = P \left( {}^B J_A \cdot {}^B \omega_{iB} \right) + {}^B \omega_{iB} \times {}^B J_A \cdot {}^B \omega_{iB} + \left( \sum_K m_K r_{AK} \right) \times {}^B P_i^2 r_{OA} \quad (5)$$

Where  ${}^B(\cdot)$  means something is defined in body coordinates frame.  ${}^B M_A$  is the whole external torques enforced to entire rotational body in proportion to  $A$  point.  ${}^B J_A$  is the entire body moment of inertia.  ${}^B \omega_{iB}$  is the velocity of body frame in proportion to inertia frame.  $r_{AK}$  is a vector from voluntary point  $A$  to point  $k$ .  $r_{OA}$  is a vector from the point  $O$  to the point  $A$ .

### III. KINEMATIC ANALYSIS AND LAYOUTS OF A 3 DEGREE OF FREEDOM ROTARY TABLE

We can apply two main kinematics and layouts for this table, which are shown in Fig. 1 and 2.

In the first kinematics, shown in Fig. 1; the outer gimbal is rotating around pitch axis, middle gimbal is rotating around yaw axis and inner gimbal is rotating around roll axis.

In the second kinematics, shown in Fig. 2; the outer gimbal is rotating around yaw axis, middle gimbal is rotating around pitch axis and inner gimbal is rotating around roll axis.

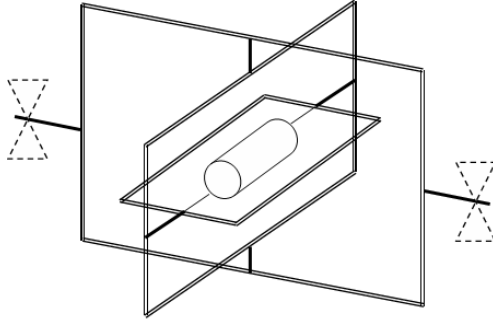


Fig. 1 – Figure of the first kinematics of 3 D.O.F Rotary Table

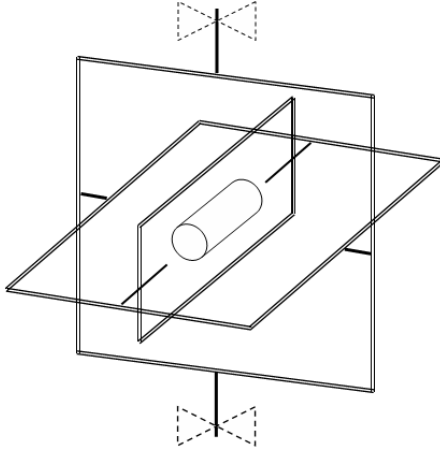


Fig. 2 – Figure of the second kinematics of 3D.O.F Rotary Table

The first kinematics has some advantages over the second one. The important advantage is that we can move the outer gimbal, which has the highest moment of inertia and weight, with two actuators; however, we move it with one actuator in the second kinematics, generally. In this case we can use smaller actuator in the first kinematics which is both cost effective and reliable for control reason. The next advantage is that we have less mechanical problems such as wobbling and unbalanced effect. On the other hand the second kinematics is theoretically in hand, whereas the original Euler's angles can be observed in the second kinematics which is not in first one. Yet we can ignore it in front of practical benefits. Thus we choose the first kinematics and define the following frames in it.

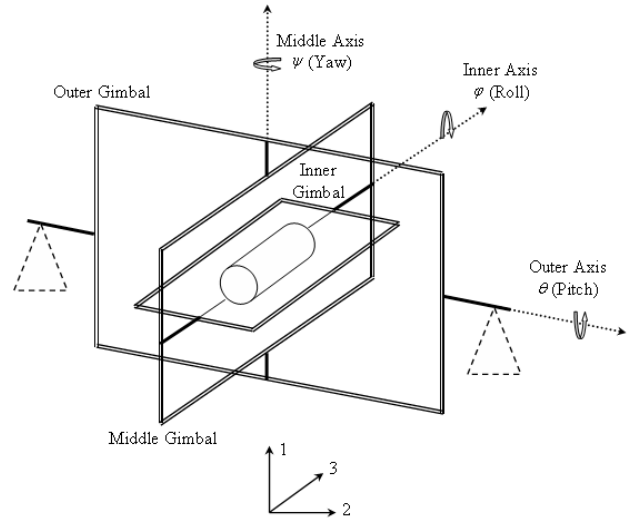


Fig. 3 – Figure of the first kinematics of A 3 D.O.F Rotary Table with its defined frames

According to Fig. 3, we defined 4 coordinates' frames as follows:

- Inner frame adhered to inner gimbal. The third axis of inner coordinates frame is in direction of inner gimbal rotation, the first one is vertical to inner gimbal surface, and the second one is the external multiplication of 2 previous axes.
- Middle frame, adhered to Middle gimbal. The third axis of middle coordinates frame is in direction of inner gimbal rotation, the first one is in direction of Middle gimbal rotation, and the second one is the external multiplication of 2 previous axes.
- Outer frame adhered to outer gimbal. The second axis of outer coordinates frame is in direction of Outer gimbal rotation, the first one is in direction of Middle gimbal rotation, and the third one is the external multiplication of 2 previous axes.
- Inertia frame adhered to the earth. The second axis of inertia coordinates frame is in direction of Outer gimbal rotation, the first one is in opposite direction of gravity, and the third one is the external multiplication of 2 previous axes.

### IV. WHOLE DYNAMIC EQUATIONS OF A 3 DEGREE OF FREEDOM ROTARY TABLE

By applying the Equation 5 to each rotational gimbal, we reach dynamic equations of a 3 Degree of Freedom Rotary Table. The dynamic equations of gimbals are calculated as follows:

#### A. Dynamic Equation of Inner Gimbal

$${}^I M_{A_i} = ({}^I J_I + {}^I J_W) \cdot P ({}^I \omega_{II}) + {}^I \omega_{II} \times ({}^I J_I + {}^I J_W) {}^I \omega_{II} + m_I \cdot {}^I r_{A_i C_i} \times {}^I P_i^2 r_{A_i O_i} + m_W \cdot {}^I r_{A_i C_W} \times {}^I P_i^2 r_{A_i O_i} \quad (6)$$

Where  $m_I \cdot {}^I r_{A_C I} \times {}^I P_i^2 r_{A_O A_I}$  and  $m_W \cdot {}^I r_{A_C W} \times {}^I P_i^2 r_{A_O A_I}$  factors are due to unbalanced and wobble error in inner gimbal and the U.U.T respectively.  ${}^I J_I + {}^I J_W$  is inner gimbal moment of inertia plus U.U.T moment of inertia,  ${}^I \omega_{II}$  is rotational velocity of inner gimbal in proportion to inertia coordinates frame,  $m_I$  is mass of inner gimbal and  $m_W$  is mass of the U.U.T.

We considered some points such as  $A_I$  which is the middle point in rotating axis of inner gimbal,  $A_O$  which is the middle point in rotating axis of outer gimbal and we consider it as an inertia point either.  $C_I$  is the inner gimbal center of mass point and finally  $C_W$  is the U.U.T center of mass point.

### B. Dynamic Equation of Middle Gimbal

$$\begin{aligned} {}^M M_{A_M} &= {}^M J_M \cdot P({}^M \omega_{iM}) + {}^M \omega_{iM} \times ({}^M J_M \cdot {}^M \omega_{iM}) \\ &+ {}^M C \cdot ({}^I J_W + {}^I J_I) \cdot P({}^I \omega_{II}) \\ &+ {}^M C \cdot {}^I \omega_{II} \times (({}^I J_W + {}^I J_I) {}^I \omega_{II}) \\ &+ m_M \cdot {}^M r_{A_M C_M} \times {}^M P_i^2 r_{A_O A_M} \\ &+ m_W \cdot {}^M r_{A_M A_I} \times {}^M P_i^2 r_{A_M C_W} + m_W \cdot {}^M r_{A_I C_W} \times {}^M P_i^2 r_{A_M A_I} \\ &+ m_W \cdot {}^M r_{A_M C_W} \times {}^M P_i^2 r_{A_O A_M} + m_I \cdot {}^M r_{A_M A_I} \times {}^M P_i^2 r_{A_M C_I} \\ &+ m_I \cdot {}^M r_{A_I C_I} \times {}^M P_i^2 r_{A_M A_I} + m_I \cdot {}^M r_{A_M C_I} \times {}^M P_i^2 r_{A_O A_M} \end{aligned} \quad (7)$$

Where the last 7 factors are due to unbalanced and wobble error in middle and inner gimbals plus U.U.T.  ${}^M J_M$  is the middle gimbal moment of inertia,  ${}^M \omega_{iM}$  is rotational velocity of middle gimbal in proportion to inertia coordinates frame,  ${}^M C$  is transformation matrix between middle coordinates frame and inner coordinates frame and  $m_M$  is the mass of middle gimbal.

We considered some points such as  $A_M$  which is middle point in rotating axis of middle gimbal and  $C_M$  which is the middle gimbal center of mass point.

### C. Dynamic Equation of Outer Gimbal

$$\begin{aligned} {}^O M_{A_O} &= {}^O J_O \cdot P({}^O \omega_{iO}) + {}^O \omega_{iO} \times ({}^O J_O \cdot {}^O \omega_{iO}) \\ &+ {}^O C \cdot {}^M J_M \cdot P({}^M \omega_{iM}) \\ &+ {}^O C \cdot {}^M \omega_{iM} \times ({}^M J_M \cdot {}^M \omega_{iM}) \\ &+ {}^O C \cdot {}^M C \cdot ({}^I J_W + {}^I J_I) {}^I P_i(\omega_{II}) \\ &+ {}^O C \cdot {}^M C \cdot {}^I \omega_{II} \times (({}^I J_W + {}^I J_I) {}^I \omega_{II}) \\ &+ m_M \cdot ({}^O r_{A_O A_M} \times {}^O P_i^2 r_{A_O C_M} + {}^O r_{A_M C_M} \times {}^O P_i^2 r_{A_O A_M}) \\ &+ m_I \cdot ({}^O r_{A_O A_I} \times {}^O P_i^2 r_{A_O C_I} + {}^O r_{A_I C_I} \times {}^O P_i^2 r_{A_O A_I}) \\ &+ m_W \cdot ({}^O r_{A_O A_I} \times {}^O P_i^2 r_{A_O C_W} + {}^O r_{A_I C_W} \times {}^O P_i^2 r_{A_O A_I}) \end{aligned} \quad (8)$$

Where the last 3 factors are due to unbalanced and wobble error in middle and inner gimbals plus U.U.T.  ${}^O J_O$  is outer gimbal moment of inertia,  ${}^O \omega_{iO}$  is rotational velocity of outer gimbal in proportion to inertia coordinates frame and  ${}^O C$  is transformation matrix between outer coordinates frame and middle coordinates frame.

We considered some point such as  $C_O$  which is the outer gimbal center of mass point.

## V. DETAILED CALCULATION IN DYNAMIC EQUATIONS OF A 3 DEGREE OF FREEDOM ROTARY TABLE

### A. Rotational Velocities Calculation

First of all we calculate the rotational velocity of inner gimbal in proportion to inertia coordinates frame, described as follows:

$${}^I \omega_{II} = {}^I C \cdot {}^M C \cdot {}^O \omega_{iO} + {}^I C \cdot {}^M \omega_{OM} + {}^I \omega_{MI} \quad (9)$$

Where

$${}^I \omega_{MI} = \begin{pmatrix} 0 \\ 0 \\ \dot{\varphi} \end{pmatrix}, {}^M \omega_{OM} = \begin{pmatrix} \dot{\psi} \\ 0 \\ 0 \end{pmatrix}, {}^O \omega_{iO} = \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} \quad (10)$$

And

$${}^M C = C_3(-\varphi) = \begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11)$$

And

$${}^O C = C_1(-\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{pmatrix} \quad (12)$$

Thus, we reach  ${}^I \omega_{II}$  according to Equation 9, 10, 11, 12:

$${}^I \omega_{II} = \begin{pmatrix} \sin \varphi \cdot \cos \psi \cdot \dot{\theta} + \cos \varphi \cdot \dot{\psi} \\ \cos \varphi \cdot \cos \psi \cdot \dot{\theta} - \sin \varphi \cdot \dot{\psi} \\ -\sin \psi \cdot \dot{\theta} + \dot{\varphi} \end{pmatrix} \quad (13)$$

Then we calculate the rotational velocity of middle gimbal in proportion to inertia coordinates frame, described as follows:

$${}^M \omega_{iM} = {}^M C \cdot {}^O \omega_{iO} + {}^M \omega_{OM} \quad (14)$$

Where

$${}^M C = C_1(-\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{pmatrix} \quad (15)$$

Thus, we reach  ${}^M \omega_{iM}$  according to Equation 10, 14, 15:

$${}^M \omega_{iM} = \begin{pmatrix} \dot{\psi} \\ \text{Cos} \psi \cdot \dot{\theta} \\ -\text{Sin} \psi \cdot \dot{\theta} \end{pmatrix} \quad (16)$$

Finally, the rotational velocity of outer gimbal in proportion to inertia coordinates frame, defines as follows:

$${}^O \omega_{iO} = \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} \quad (17)$$

### B. External Torques Calculation

We have external torques in inner gimbal, middle and outer one respectively,

$${}^I M_{A_I} = \begin{pmatrix} T_{1MI} \\ T_{2MI} \\ T_{eI} - f_I \cdot \dot{\varphi} \end{pmatrix} \quad (18)$$

$${}^M M_{A_M} = \begin{pmatrix} T_{eM} - f_M \cdot \dot{\psi} \\ T_{2OM} \\ T_{3OM} \end{pmatrix} \quad (19)$$

$${}^O M_{A_O} = \begin{pmatrix} T_{iO} \\ T_{eO} - f_O \cdot \dot{\theta} \\ T_{3iO} \end{pmatrix} \quad (20)$$

Where  $T_{eI}$  is electromagnetic torque of inner gimbal actuator,  $T_{eM}$  is electromagnetic torque of middle gimbal actuator and  $T_{eO}$  is electromagnetic torque of outer gimbal actuator.  $f_I$  is friction coefficient between inner gimbal and middle gimbal,  $f_M$  is friction coefficient between middle gimbal and outer gimbal and  $f_O$  is friction coefficient between outer gimbal and stand.

### C. Moments of Inertia Calculation

If we chose the coordinates frames in a way that their axes adhere to each gimbal axes of symmetry, then we will receive to diagonal moment of inertia matrix, shown below,

$${}^I J_{W+I} = \begin{pmatrix} J_{W11} + J_{I11} & 0 & 0 \\ 0 & J_{W22} + J_{I22} & 0 \\ 0 & 0 & J_{W33} + J_{I33} \end{pmatrix} \quad (21)$$

$${}^M J_M = \begin{pmatrix} J_{M11} & 0 & 0 \\ 0 & J_{M22} & 0 \\ 0 & 0 & J_{M33} \end{pmatrix} \quad (22)$$

$${}^O J_O = \begin{pmatrix} J_{O11} & 0 & 0 \\ 0 & J_{O22} & 0 \\ 0 & 0 & J_{O33} \end{pmatrix} \quad (23)$$

Consequently, we reach detailed dynamic equations as follows:

If we can overcome the unbalanced and wobbles errors in middle and inner gimbals plus U.U.T, we will reach to the following equations; nevertheless, we have the unbalanced and wobble errors equations in hand:

$$\begin{aligned} T_{eI} - f_I \cdot \dot{\varphi} &= (J_{W33} + J_{I33}) \\ &\cdot \left( -\text{Cos} \psi \cdot \dot{\psi} \cdot \dot{\theta} - \text{Sin} \psi \cdot \ddot{\theta} + \ddot{\varphi} \right) \\ &+ \left[ (J_{W22} + J_{I22}) - (J_{W11} + J_{I11}) \right] \\ &\cdot \left( \text{Sin} \varphi \cdot \text{Cos} \psi \cdot \dot{\theta} + \text{Cos} \varphi \cdot \dot{\psi} \right) \\ &\cdot \left( \text{Cos} \varphi \cdot \text{Cos} \psi \cdot \dot{\theta} - \text{Sin} \varphi \cdot \dot{\psi} \right) \end{aligned} \quad (24)$$

$$\begin{aligned} T_{eM} - f_M \cdot \dot{\psi} &= \\ &\left[ (J_{W11} + J_{I11}) \cdot \text{Cos}^2 \varphi + (J_{W22} + J_{I22}) \cdot \text{Sin}^2 \varphi \right] \\ &\cdot \left( \text{Cos} \psi \cdot \dot{\varphi} \cdot \dot{\theta} + \ddot{\psi} \right) + \left[ (J_{W11} + J_{I11}) - (J_{W22} + J_{I22}) \right] \\ &\cdot \text{Sin} \varphi \cdot \text{Cos} \varphi \cdot \left( -\text{Sin} \psi \cdot \dot{\psi} \cdot \dot{\theta} + \text{Cos} \psi \cdot \ddot{\theta} - \dot{\varphi} \cdot \dot{\psi} \right) \\ &+ \left[ (J_{W33} + J_{I33}) - (J_{W22} + J_{I22}) \right] \cdot \left( -\text{Sin} \psi \cdot \dot{\theta} + \dot{\varphi} \right) \\ &\cdot \left( \text{Cos}^2 \varphi \cdot \text{Cos} \psi \cdot \dot{\theta} - \text{Sin} \varphi \cdot \text{Cos} \varphi \cdot \dot{\psi} \right) \\ &\left[ (J_{W33} + J_{I33}) - (J_{W11} + J_{I11}) \right] \cdot \left( -\text{Sin} \psi \cdot \dot{\theta} + \dot{\varphi} \right) \\ &\cdot \left( \text{Sin}^2 \varphi \cdot \text{Cos} \psi \cdot \dot{\theta} + \text{Sin} \varphi \cdot \text{Cos} \varphi \cdot \dot{\psi} \right) + J_{M11} \cdot \ddot{\psi} \\ &+ (J_{M22} - J_{M33}) \cdot \text{Sin} \psi \cdot \text{Cos} \psi \cdot \dot{\theta}^2 \end{aligned} \quad (25)$$

Detailed equation of Outer Gimbal is so large to insert in the paper.

### D. Contribution

If we construct the inner gimbal in such a way that the moment of inertia matrix of inner gimbal plus U.U.T is diagonal and the elements of this matrix were equal, shown in Equation 26, we will reach simple and minimized dynamic equation of inner gimbal shown in Equation 27. This is, of course, without consideration of unbalanced and wobble error.

$$(J_{W11} + J_{I11}) = (J_{W22} + J_{I22}) = (J_{W33} + J_{I33}) = M_I \quad (26)$$

$$T_{eI} - f_I \cdot \dot{\varphi} = -M_I \cdot \text{Cos} \psi \cdot \dot{\psi} \cdot \ddot{\theta} - M_I \cdot \text{Sin} \psi \cdot \ddot{\theta} + M_I \cdot \ddot{\varphi} \quad (27)$$

Similarly, if we cover the Equation 28 and 29 in constructions of middle and outer gimbals, we will reach the simple and minimized dynamic equations of them as below,

$$J_{M11} = J_{M22} = J_{M33} = M_M \quad (28)$$

$$J_{O11} = J_{O22} = J_{O33} = M_O \quad (29)$$

$$T_{eM} - f_M \cdot \dot{\psi} = 2 \cdot M_I \cdot \text{Cos} \psi \cdot \dot{\varphi} \cdot \ddot{\theta} + (2 \cdot M_I + M_M) \cdot \ddot{\psi} \quad (30)$$

$$T_{eO} - f_O \cdot \dot{\theta} = -M_I \cdot \text{Sin} \psi \cdot \ddot{\varphi} - M_I \cdot \text{Sin} \psi \cdot \text{Cos} \psi \cdot \dot{\psi} \cdot \ddot{\theta} - 2 \cdot M_I \cdot \text{Cos} \psi \cdot \dot{\varphi} \cdot \dot{\psi} \quad (31)$$

$$\left[ (M_I + M_M) \cdot \text{Sin}^2 \psi + (2 \cdot M_I + M_M) \cdot \text{Cos}^2 \psi + M_O \right] \cdot \ddot{\theta}$$

## VI. VERIFICATION OF RESULTED DYNAMIC EQUATIONS

Now, we verify the obtained dynamic equations of a 3 Degree of Freedom Rotary. Consider that we want do design a table with the following sizes approximately,

- Outer gimbal outer size:  $2.2^m \times 2.2^m \times 0.2^m$ .
- Middle gimbal outer size:  $1.8^m \times 1.8^m \times 0.2^m$ .
- And Inner gimbal outer size:  $1.4^m \times 1.4^m \times 0.2^m$ .

If we consider magnesium as the body's material with density of about  $1800 \frac{\text{kg}}{\text{m}^3}$ , we will reach the following moments of inertia for the gimbals.

$$M_I = 50 \text{ml}^2, M_M = 120 \text{ml}^2, M_O = 220 \text{ml}^2 \quad (32)$$

Verification of the equations with MATLAB software is displayed in the following figures. Input of each gimbal is unit step electromagnetic torque.

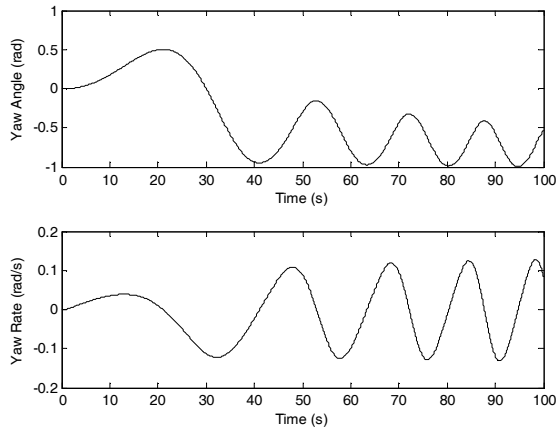


Fig. 4 – Yaw angle and yaw rate

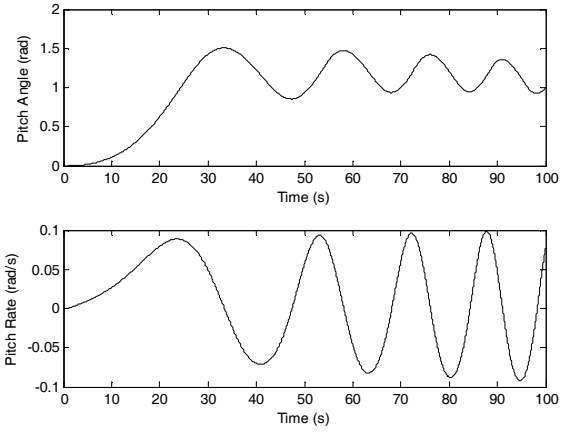


Fig. 5 – Pitch angle and pitch rate

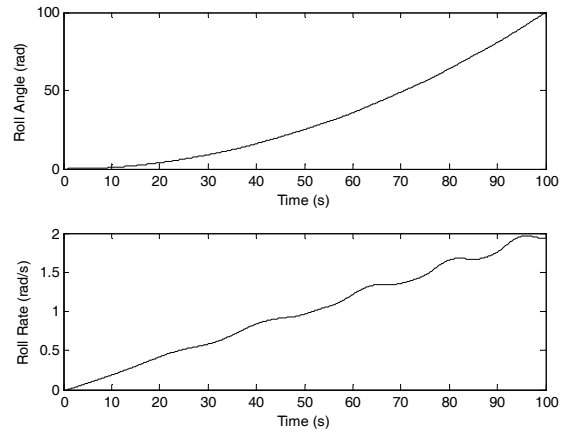


Fig. 6 – Roll angle and roll rate

If we review the Equation 27, we will find that it contains the following terms,

- $M_I \cdot \ddot{\varphi}$  which is a rotational torque due to apparent acceleration of inner gimbal in proportion to middle one.
- $-M_I \cdot \text{Sin} \psi \cdot \ddot{\theta}$  which is a rotational torque that contains apparent acceleration of outer gimbal with  $\text{Sin} \psi$  coefficient. It shows that if the middle gimbal turns  $90^\circ$ , the inner gimbal will conform to outer one and accept outer gimbal apparent acceleration.
- $-M_I \cdot \text{Cos} \psi \cdot \dot{\psi} \cdot \ddot{\theta}$  which is a Coriolis effect Torque due to outer gimbal velocity ( $\ddot{\theta}$ ) and middle gimbal velocity ( $\dot{\psi}$ ).

These 2 previous terms bring about oscillations as we see in Fig. 6.

If we review the Equation 30, we will find that it contains the following terms,

- $(2 \cdot M_I + M_M) \cdot \ddot{\psi}$  which is a rotational torque due to apparent acceleration of middle gimbal in proportion to outer one.
- $2 \cdot M_I \cdot \cos \psi \cdot \dot{\varphi} \cdot \dot{\theta}$  which is a Coriolis effect Torque due to outer gimbal velocity ( $\dot{\theta}$ ) and inner gimbal velocity ( $\dot{\varphi}$ ).

The last term brings about oscillation as we see in Fig. 4.

If we review the Equation 31, we will find that it contains the following terms,

- $\left[ (M_I + M_M) \cdot \sin^2 \psi + (2 \cdot M_I + M_M) \cdot \cos^2 \psi + M_O \right] \cdot \ddot{\theta}$  which is a rotational torque due to apparent acceleration of outer gimbal in proportion to the stand of table.
- $-M_I \cdot \sin \psi \cdot \ddot{\varphi}$  which is a rotational torque that contains apparent acceleration of inner gimbal with  $\sin \psi$  coefficient. It shows that if the middle gimbal turns  $90^\circ$ , the outer gimbal will conform to inner one and accept inner gimbal apparent acceleration.
- $-M_I \cdot \sin \psi \cdot \cos \psi \cdot \dot{\psi} \cdot \dot{\theta}, -2 \cdot M_I \cdot \cos \psi \cdot \dot{\varphi} \cdot \dot{\psi}$  are Coriolis effect Torques, due to the outer gimbal velocity ( $\dot{\theta}$ ), middle gimbal velocity ( $\dot{\psi}$ ) and inner gimbal velocity ( $\dot{\varphi}$ ).

These last two terms bring about oscillations as we see in Fig. 5.

## VII. CONCLUSIONS

We calculated full dynamic equations of a 3 Degree of Freedom Rotary Table which contain all designing parameters, even the unbalanced and wobble effect, by applying the Newton's second law for rotation.

In this paper we showed that it is better to construct the inner and middle gimbals in such a way that their moment of inertia matrix is coefficient of unit matrix in order to reach simple and minimized dynamic equations that is useful in 3 Degree of Freedom Rotary Table's control. If it is not possible, we can use these detailed equations to have an efficient control strategy.

According to these detailed equations we find all rotational torques that enforce to the bearings in any direction. First, we can use them to choose a proper passive ball bearing. Second, we can use them to construct a proficient active bearing such as magnet bearing or air bearing instead of passive one.

## REFERENCES

- [1] Dorosti M. (2007) 'Designing 3 Degree of Freedom Rotary Table', Bachelor's Thesis in Electrical Engineering-control, K. N. Toosi University of Technology
- [2] Nobari J. H. (2006) "Discussion in Flying and guidance", K. N. Toosi University of Technology
- [3] Nobari J. H. (2006) "Discussion in Navigation Systems", K. N. Toosi University of Technology
- [4] Alasti A., Abedi H., 'Kinematic and Dynamic Sensitivity Analysis of a Three-Axis Rotary Table', proceeding of IEEE Conference on Control Application, CCA 2003, and Paper Identity code: CD-000741.
- [5] L.A.Demore, et all, 'Design Study fore High-Accuracy Three Axis Test Table', *Journal of Guidance*, Vol.10, No.1, pp.104-114, Jan-Feb.1987.
- [6] A. Bedford and W. Fowler, "Engineering Mechanics Dynamics", Addison-Wesley Publishing Co, 1995
- [7] Christopher Jekeli, "Inertial Navigation Systems with Geodetic Applications", Walter de Gruyter, Berlin, New York 2001.
- [8] R. C. Hibbeler, "Engineering Mechanics Dynamics", Mc Millan Publishing Co, New York, 1992